Uttar Pradesh Rajarshi Tandon
Open University

Postgraduate Diploma in Computer Application

PGDCA-107

Data Structure

RIL-102

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BLOCK

1

Introduction of Data Structures, Basics of Algorithms and Array

Curriculum Design Committee

Dr.P.P.Dubey — **Coordinator**
Director, School of Agri. Sciences,
UPRTOU, Prayagraj

Prof. U. N. Tiwari — **Member**
Dept. of Computer Science and Engg.,
Indian Inst. Of Information Science and Tech.,
Prayagraj

Prof. R.S. Yadav, — **Member**
Dept. of Computer Science and Engg.,
MNNIT, Allahabad, Prayagraj

Prof. P. K. Mishra — **Member**
Dept. of Computer Science,
Baranas Hindu University, Varanasi

Mr. Prateek Kesrwani — **Member Secretary**
Academic Consultant-Computer Science
School of Science, UPRTOU, Prayagraj

Course Design Committee

Prof. U. N. Tiwari — **Member**
Dept. of Computer Science and Engg.,
Indian Inst. Of Information Science and Tech.,
Prayagraj

Prof. R.S. Yadav, — **Member**
Dept. of Computer Science and Engg.,
MNNIT, Allahabad, Prayagraj

Prof. P. K. Mishra — **Member**
Dept. of Computer Science,
Baranas Hindu University, Varanasi

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Dr. S . S . T ripathi, A cademic C onsultant (Maths), School of S cience, UPRTOU, Prayagraj

Course Preparation Committee

Prof. Manu Pratap Singh, **Author**
Dept. of Computer Science
Dr. B. R. Ambedkar University, Agra-282002

Dr. Ashutosh Gupta **Editor**
Director, School of Sciences,
UPRTOU, Prayagraj

Prof. U. N. Tiwari **Member**
Dept. of Computer Science and Engg.,
Indian Inst. Of Information Science and Tech.,
Prayagraj

Prof. R.S. Yadav **Member**
Dept. of Computer Science and Engg.,
MNNIT, Allahabad, Prayagraj

Prof. P. K. Mishra **Member**
Dept. of Computer Science
Baranas Hindu University, Varanasi

Dr. Dinesh K Gupta, **SLM Coordinator**
Academic Consultant- Chemistry School of Science, UPRTOU, Prayagraj

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COURSE INTRODUCTION

The objective of this course is to introduce the basic concepts of data structures. The implementation details of various types of data structures and their applications for the computations and in computer science for solving the real world problems. Various types of data structures like linear and non-linear are discussed in detail. After reading this course material you are able to understand the meaning of data structure and also learn about their implementation. You will also able to see how the data structures useful for the computation. The aim is to provide an extensive variety of topics on this subject with appropriate examples. The course is organized into following blocks:

Block-1 Introduction of data structures, basics of algorithms and Array

Block-2 Data structures like Stack & Queue and recursion.

Block-3 Linked list, Tree and Graph.

Block-4 Searching & Sorting, Hashing and File organization

UNIT-1 INTRODUCTION TO DATA STRUCTURE

Structure

1.0 INTRODUCTION

This unit is an introductory unit and gives you an understanding of data structure, Algorithm, Data representation, various Data types and a general overview about linear and non-linear data structures. It is about structuring and organizing data as a fundamental aspect of developing a computer application.

1.1 OBJECTIVES

After the end of this unit, you should be able to:

1. Understand about algorithm.
2. Understand of the data organization and representation
3. Define the term data structure
4. Understand about various data types
5. Know the classification of data structure i.e. linear and non-linear
6. Introduce with data structure representation and operation on data structures

1.2 ALGORITHM DEFINITION

An algorithm is a set of instructions to be done sequentially. Any work to be done can be thought as series of steps. For example, to perform an experiment, one must do some sequential tasks like:

1. Set up the required apparatus
2. Do the process required
3. Note any observations
4. Summarize the results

These tasks accomplish an experiment. Let us see where such sequential steps are employed.

A computer or other electronic device which accomplishes a logical task is not actually logical but is simply following a series of programmed, sequential instructions. A computer algorithm consists of a series of well-defined steps given to the computer to follow. We can also define the algorithm as:

1. "An algorithm is a well define procedure that takes some value as input and produces some value as the output in finite number of steps."
2. "An algorithm is thus a sequence of computational steps that transform the input into the output that must halt after a final number of steps or time"
3. "An algorithm is a procedure for processing that is formulated so precisely that it may be performed by a mechanical or electronic devices must be formulated so exactly that the sequence of the processing steps is completely clear and it has to terminate in definite time."

The typical examples for algorithms are computer programs written in a formal programming language.

Example: Write an algorithm to exchange the value of two variables.

Algorithm:

1. Consider the two variables a, b with some values.

2. Consider a third variable t.

3. Do the following steps

(a) Assign the value of a to t

(b) Assign the value of b to a

(c) Assign the value of t to b

4. Print the exchange value of a and b

1.3 BASIC CRITERIA FOR ALGORITHMS:

There are following basic criteria for an algorithm:

1. The algorithm must be expressed in a fashion that is completely free of ambiguity.
2. The algorithm must be efficient. It should not unnecessarily use memory locations nor should it require an excessive number of logical operations.
3. The algorithms should be concise and compact to facilitate verification of their correctness.
4. The algorithm must be independent from any programming language mostly its written in pseudo code, so it can convert to any programming language with the proper use of programming language syntax.

1.4 DATA STRUCTURE DEFINITION

First we define the meaning of simple data. Data are simply values or set of values. A data item is either the value of a variable or a constant. For example, the value of a variable x is 5 which is described by the data type integer, a data item is a row in a database table, which is described by a data type. A data item that does not have subordinate data items is called an elementary item. A data item that is composed of one or more subordinate data items is called a group item. A record can be either an elementary item or a group item. For example, an employee's name may be divided into three sub items – first name, middle name and last name but the social_security_number would normally be treated as a single item. Data may be organized in many different ways:

- The logical or mathematical model of a particular organization of data is called a data structure.
- A data structure is an arrangement of data in a computer's memory or even disk storage.
- Data structure is the method to store and organize data to facilitate access and modifications
- A data structure, sometimes called data type, can be thought of as a category of data. Like, Integer is a data category which can only contain integers. String is data category holding only strings. A data structure not only defines what elements it may contain, it also supports a set of operations on these elements, such as addition or multiplication.
- Data structures are ways to organize data (information) for example:
    - Simple variables are consider as the Primitive types
    - Array, the collection of data items of the same type, stored in memory at contiguously

- Linked list, the sequence of data items, each one points to the next one, stored in memory at non-contiguously.

- Data structures are building blocks of a program. If program is built using improper data structures, then the program may not work as expected always.

- The possible ways in which the data items are logically related define different data structures.

- A data structure is a collection of different data items that are stored together in a clearly defined way.

The examples of several common data structures are **string, arrays, Stacks, Queues, Linked list, Binary Trees, Graph and Hash Tables.**

In combination with Algorithm we may define the data structures as:

- Algorithms go with the data structures to manipulate the data i.e. Algorithms are used to manipulate the data contained in these data structures as in the form of sorting and searching.

- More generally we can say: **Algorithms + Data Structures = Programs.**

1.5 DATA TYPE

A data type is a classification of data, which can store a specific type of information. Data types are primarily used in computer programming, in which variables are created to store data. Each variable is assigned a data type that determines what type of data the variable may contain. Thus a data type is a method of interpreting a pattern of bits. There are numerous different data types. They are used to make the storage and processing of data easier and more efficient.

A data type is a term which refers to the kinds of data that variables may hold. With every programming language there is a set of built-in data types. This means that the language allows variables to name data of that type and provides a set of operations which meaningfully manipulates these variables. Some data types are easy to provide because they are built-in into the computer's machine language instruction set, such as integer, character etc. Other data types require considerably more efficient ways to implement. In some languages, these are features which allow one to construct combinations of the built-in types (like structures in 'C'). However, it is necessary to have such mechanism to create the new complex data types which are not provided by the programming language. The new type also must be meaningful for manipulations. Such meaningful data types are referred as abstract data type.

Different programming languages have their own set of basic data types.

Basic data types or primitive data types

The most common basic or intrinsic data types or primitive data types are as follows:

- Integer : It is a positive or negative number that does not contain any fractional part.
- Real : A number that contains the decimal part
- Boolean : It is a data type that can store one of only two values, usually these values are TRUE or FALSE
- Character : It is any letter, number, punctuation mark or space, which takes up a single unit of storage, usually a byte
- String : It is sometimes just referred to as ' text'. Any type of alphabetic or numeric data can be stored as a string: "Delhi City", "30/05/2013" and "459.78" are all examples of the strings. Each character within a string will be stored in one byte using its ASCII code. The maximum length of a string is limited only by the available memory.

Structure data types or Non Primitive data types

There is another class of data types which is considered as structure data types or non primitive data types. These data types are user defined data types. Structured data types hold a collection of data values. This collection will generally consist of the primitive data types. Examples of this would include arrays, records, list, tree and files. These data types, which are created by programmers, are extremely important and are the building block of data structures. These are more complex data structures. They stress on formation of sets of homogeneous and heterogeneous data elements.

Abstract data types or Non-primitive data types

It is another form of the non-primitive data types. An abstract data type can be assumed as a mathematical model with a collection of operations defined on that model i.e. an Abstract data type (ADT) is a new data type derived or created from basic or built in data type based on a particular logical or mathematical model. For example Set of integers consisting of different numbers may be an ADT. A set is a combination of more than one integer, but the operations on set is a generalized operation of different integers such as union, intersection, product, and difference.

Note: If the data contains a single value this can be organized using primitive data type. If the data contains set of values they can be represented using non-primitive data types.

Now on the basis of above mentioned data types the data structure can be defined as:

"*An implementation of abstract data type is data structure i.e. a mathematical or logical model of a particular organization of data is called data structure.*"

1.6 TYPE OF DATA STRUCTURES

A data structure is the portion of memory allotted for a model, in which the required data can be arranged in a proper fashion. Normally The data structures are of two types or it can be broadly classified into two types of data structures:

(i) Primitive data structure

(ii) Non-primitive data structure

1. Primitive data structure : The data structures that typically are directly operated upon by machine level instruction. Examples: Integers, Real numbers, Characters and pointers, and their corresponding storage representation. Programmers can use these data types when creating variables in their programs. For example, a programmer may create a variable say "z" and define it as a real data type. The variable will then store data as a real number.

2. Non primitive data structure : Non – primitive data structures are not defined by the programming language, but are instead created by the programmer. They are also called as the reference variables, since they reference a memory location, which stores the data. These data structures are derived from the primitive data structures. Examples: Arrray, Stack, Queues, Linked list, Tree, Graphs and hash table.

   There are two type of-primitive data structures.

   a) Linear Data Structures:-

   In linear data structure the elements are stored in sequential order. Hence they are in the form of a list, which shows the relationship of adjacency between elements and is said to be linear data structure. The most, simplest linear data structure is a 1-D array, but because of its deficiency, list is frequently used for different kinds of data. The linear data structures are:

   (i) Array : The Array is a collection of data of same data type stored in consecutive memory location and is referred by common name.

   (ii) Stack : A stack is a Last-In-First-Out or First-In-Last-Out linear data structure in which insertion and deletion takes place at only from one end called the top of the stack.

(iii) Queue : A Queue is a First-In-First-Out or Last-In-Last-Out data structure in which insertion takes place from one end called the rear and the deletions takes place at one end called the Front.

(iv) Linked List : Linked list is a collection of data of same type but the data items need not be stored in consecutive memory locations. It is linear but non-contiguous type data structure. A linked list may be a single list or double list.

- **Single Linked list:** - A single list is used to traverse among the nodes in one direction.
- **Double linked list:** - A double linked list is used to traverse among the nodes in both the directions.

b) Non-linear data structure:-

In non linear data structure the elements are stored based on the hierarchical relationship among the data. A list, which doesn't show the relationship of adjacency between elements, is said to be non-linear data structure. The non-linear data structures are:

(i) Tree: This data structure is used to represent data that has some hierarchical relationship among the data elements. Thus, it maintains hierarchical relationship between various elements.

(ii) Graph: This data structure is used to represent data that has relationship between pair of elements not necessarily hierarchical in nature. It maintains random relationship or point-to-point relationship between various elements. For example electrical and communication networks, airline routes, flow chart and graphs for planning projects.

1.7 REPRESENATION OF DATA STRUCTURES

There are generally two common methods for the data structure representation. These methods can be specified as:

(i) Sequential representation

(ii) Linked representation

(i) Sequential representation

A sequential representation maintains the data in continuous memory locations which takes less time to retrieve the data but leads to more time during insertion and deletion operations due to its sequential nature.

(ii) Linked Representation

Linked representation maintains the list by means of a link between the adjacent elements which need not be stored in continuous memory locations. During insertion and deletion operations, links will be created or removed between which it takes less time when compared to the corresponding operations of sequential representation. Generally, linked representation is preferred for any data structure.

1.8 DATA STRUCTURE OPERATIONS

The data elements appearing in the data structure is processed by means of certain operations. In fact the particular data structure that one chooses for a given situation depends largely on the frequency with which specific operations are performed. The following major operations performed on data structures are:

Insertion

It provides the means for adding new details or new node into the existing data structure.

Deletion

It provides the means for removing a node from the data structure.

Traversing

It provides the means for accessing each node exactly once so that the nodes of a data structure can be processed. It is also called the visiting to data structure.

Searching

It provides the means for finding the location of node for a given key value or finding the locations of all records, which satisfy one or more conditions.

Sorting

It provides the means for arranging the data in a particular order in given data structure.

Merging

It provides the means for joining the two data structures.

Note : Sometimes two or more data structure of operations may be used in a given situations; e. g. we may want to delete the records with a given key, which may means we first need to search for the location of the record.

1.9 SUMMARY

- Data structure is the particular organization of data either in a logical or mathematical manner
- Data type is a concept that defines internal representation of data.
- Data structures are building blocks of a program. If program is built using improper data structures, then the program may not work as expected always
- An algorithm is a set of instructions to be done sequentially. Any work to be done can be thought as series of steps.
- An abstract data type is the specification of logical and mathematical properties of data types or structure. It acts as a guideline to implement a data structure.
- A data structure is the portion of memory allotted for a model, in which the required data can be arranged in a proper fashion
- Primitive and non-primitive are the two basic data types of data structure.
- The relationship between abstract data type and data structure is well defined. An abstract data type is the specification of a data type whereas data type is the implementation of abstract data type and data structure comprises computer variable of same or different data types.
- There are generally two common methods for the data structure representation i.e. Sequential and linked representation.

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SELF EVALUATION

1. Define the data structure and Algorithm.
2. What are different data types? Give the example of each data type.
3. Specify the types of data structure with the example of each type.
4. What are the various operations on data structures?
5. While Considering the data structure implementations, the factor under consideration is/are:
   a. Time
   b. Space and Time
   c. Time, Space and Processor
   d. None of the above
6. A data type is the collection of values and the set of operations on values (True/False)
7. ……..refers to the collection of computer variables that are connected in some specific manner.
8. One of the example of a structured data type can be……..
9. Explain abstract data type with an example.
10. What is a data structure and what are the difference between data types, abstract data type and data structure?
11. An ----------data type is a keyword of a programming language that specifies the amount of memory needed to store data and the kind of data that will be stored in that memory location.
    a. Abstract b. int
    c. vector d. None of these
12. Graphs are classified into …….. category of data structure.
13. What do you mean by LIFO and FIFO?

UNIT-2 BASICS OF ALGORITHM

Structure

2.0 INTRODUCTION

This unit is an introductory unit about the algorithm basics and complexity of the algorithm. It gives you an understanding about Algorithm structure, the format for writing the algorithm and the time of execution and space in memory during the course of execution for the algorithm which considers as the time and space complexity of the algorithm.

2.1 OBJECTIVES

Algorithm designing is an important process of solving the problem. The designing of an algorithm considers various aspects like space and time requirement for the execution of algorithm. At the end of this unit, you will be able to:

1. Understand about basic of algorithm.
2. Understand of Computation time for execution of algorithm
3. Understand about requirement of the space during the course of execution for algorithm.
4. Notation for determining the time complexity of the algorithm (Asymptotic notations)
5. Understand the analysis of algorithm with asymptotic notations

2.2 ALGORITHM

The algorithm provides the way for solving the given problem in a systematic way. The term algorithm refers to the sequence of instructions that must be followed to solve a problem. Alternatively, an algorithm is a logical representation of the instructions which should be executed to perform a meaningful task. There are following characteristics of the algorithm:

- Each instruction should be unique and concise
- Each instruction should be relative in nature and should not repeat infinitely
- Results should be available to the user after the algorithm terminates.

Therefore, an algorithm is any defined computational procedure, along with a specified set of allowable inputs that produce some value or set values as output. There are two basic approaches for designing the algorithm.

- Top-Down approach: In this approach we start from the main component of the program and decomposing it into sub problem or components. This process continues until all the sub modules do not solve. Top-down design method takes the form of **stepwise refinement**. In this, we start with the topmost module and incrementally add modules that it calls.
- Bottom – Up approach: In this approach of designing we start with the most basic or primitive components and proceeds to higher-level components. Bottom-up method works with **layer of abstraction**.

Here a simple example of the algorithm is presented to demonstrate the various algorithmic notations and a way to express the algorithm for solving the problem.

Example :

Algorithm Greatest:

This algorithm finds the largest algebraic element of vector A which contains N elements and places the result in MAX. I is used to subscript A.

1. [Is the vector empty?]

   If N < 1

   Then print('Empty Vector')

   Exit

2. [Initialize]

   MAX=A[1] [We assume initially that A[1] is the greatest element]

   I=2

3. [Examine all elements of vector]

   Do while I<=N

   3.1 [Change MAX if it is smaller than the next element]

   If MAX < A[I]

   Then MAX= A[I]

   3.2 [Prepare to examine next element in vector]

   I = I+1

4. [Finished]

   Exit

2.3 FORMAT CONVENTION OF ALGORITHM

Hence from this example we can consider the following format conventions for writing the algorithm. These conventions are so general that these may be used for writing any algorithm.

- Name of Algorithm: Every algorithm is given an identify name
- Introductory Comment: The algorithm name is followed by a brief description of the tasks the algorithm performs.
- Steps: The algorithm is made up of a sequence of numbered steps, each beginning with a phrase enclosed in square brackets which gives an abbreviated description of that step.
- Comments: Every step of the algorithm is explained for better understanding of it. These comments are expressed in brackets. Comments specify no action and are included only for clarity.
- Statements and control Structures: It includes the various operators and looping methods those are required for logical and arithmetical operations. For example; Assignment statement, If-statement, Case statement and looping methods.
- Variable names: An entity that possesses a value and its name is chosen to reflect the meaning of the value it holds. For example The MAX is considered as the variable in our previous example of algorithm Greatest.
- Data structures: various data structures including static and dynamic structures are used for the implementation of algorithm.

- Arithmetic operations and expressions: The algorithm notation includes the standard binary and unary operators according to their standard mathematical order of precedence as follows:

	Operation	Symbol	Order of Evaluation
1.	Parentheses	()	Inner to outer
2.	Exponentiation, Unary plus, minus	∧, ++ , _ _	Right to left
3.	Multiplication, Division	*, /	Left to right
4.	Addition, Subtraction	=, -	Left to right

- Relations and Relational Operators: There are standard relational operators (<, <=, >=, ≠, =, ==) are used with their usual meaning in the implementation of algorithm. A relation evaluates to a logical expression that is, it has one of two possible values, *True* or *False*.
- Logical operations and Expressions: The algorithmic notation also includes the standard logical operators like NOT, OR & AND with their usual meaning. These may be used to connect relations to form compound relations whose only values are *True* or *False*. In order that logical expressions be clear, we consider that operators precedence is as follows:

Precedence	Operator
1	Parentheses
2	Arithmetic
3	Relational
4	Logical

- Input and output: The algorithm notation must include the notation for input and output. The input is obtained and placed in a variable and output is obtained by getting the value from variable.

- Functions: A function is used when we want a single value returned to the calling routine. Transfer of control and returning of the value are accomplished by 'Return (value)'.
- Procedures: A procedure is similar to a function but there is no value returned explicitly. A procedure is also invoked differently, where there are parameters, a procedure returns its results through the parameters.

All algorithms must satisfy the following criteria.

1. Input
2. Output
3. Definiteness
4. Finiteness
5. Effectiveness

The criteria 1 & 2 require that an algorithm produces one or more outputs & have zero or more input. According to criteria 3, each operation must be definite such that it must be perfectly clear what should be done. According to the 4th criteria algorithm should terminate after a finite number of operations. According to 5th criteria, every instruction must be very basic so that it can be carried out by a person using only pencil & paper.

After an algorithm has been designed its efficiency must be analysed. This involves determining whether the algorithm is economical in the use of computer resources, i.e. **CPU time** and **memory requirement.** The term used to refer to the memory required by an algorithm is **memory space** and the term used to refer to the computational time is the **execution time**. The importance of efficiency of an algorithm is the **correctness**. Thus, it always produces the correct result and **algorithm complexity** which considers both the difficulty of implementing an algorithm along with its efficiency.

Therefore the requirement for implementation of an algorithm with correctness considers many aspects. The fundamental question arises is that "How can we judge how useful a certain combination of data structures and algorithm is?" Of course the answer of this question depends that how can we evaluate the effort that arises from performing a computation using the certain combination of data structures and algorithms. There may be many algorithms devised for an application and we must analyse and validate the algorithms to judge the suitable one.

Hence this effort is measured normally with following two important factors those have the direct relationship with the performance of the algorithm:

- Memory space used i.e. **Space complexity.** The space complexity of an algorithm is the amount of memory it needs to run.

- CPU time involves runtime or execution time for the program based on the algorithm i.e. **Time Complexity.** The time complexity of an algorithm is given by the number of steps taken by the algorithm to compute the function it was written for.

2.4 COMPLEXITY OF ALGORITHM

The Complexity of algorithm is considered actually as in the form of computational complexity. Computational complexity is a characterization of the time or space requirements for solving a problem by a particular algorithm. These requirements are expressed in terms of a single parameter that represents the size of the problem. For example we consider a problem of size *n.* Let the time required of a specific algorithm for solving this problem is expressed by a function:

$$f: R \longmapsto R$$

Such that *f(n)* is the largest amount of time needed by the algorithm to solve the problem of size n. The function *'f'* is usually called the time complexity function. Thus, we can say that the analysis of the algorithm requires two main considerations:

- Time Complexity
- Space Complexity

The time complexity of an algorithm is the amount of computer time that it needs to run to completion. The space of an algorithm is the amount of memory that it needs to run to completion.

2.5 TIME COMPLEXITY

In order to compute the time complexity of a n algorithm we consider only the frequency count of the important steps or instructions. Since these data structures are so widespread, it is important to implement them efficiently. This efficiency is measured using the following two methods:

- Asymptotic Analysis
- Big-O analysis

It is very general that the actual time (wall-clock time) of a program is affected by:

- Size of the input
- Programming language
- Programming tricks
- Compiler

- CPU Speed
- Multiprogramming level

Hence instead of wall clock time for the program if we consider the pattern of the program's behaviour as the program size increases. This is called the **Asymptotic Analysis.**

Big- O Analysis

If *f(n)* represent the computing time of some algorithm and *g(n)* represents a known standard function like *n, n^2, n log n etc* then to write: *f(n) is O g(n)* means that *f(n)* of *n* is equal to biggest order of function *g(n).* This implies only when:

$f(n) \leq Clog(n)$ for all sufficiently large integers *n*, where *C* is the constant. Thus from the above statements we can say that the computing time of an algorithm is *O(g(n)),* we mean that its execution time is no more than a constant time *g(n),* n is the parameter which characterizes the input and / or outputs. From the practical point of view, we get the Big-O notation for a function by:

1. Ignoring multiplicative constants (these are due to pesky difference in compiler, CPU, etc.)
2. Discarding the lower order terms (as *n* gets larger, the largest term has the biggest impact). Like;
   - *8410 = O(1)*
   - $100n^3+nlogn+67n^7+4n = O(n^7)$

The Big-O notation helps to determine the time as well as space complexity of the algorithms. The Bog-O notation has been extremely useful to classify algorithms by their performances. Now we consider the three simple algorithms with different number of sequences or steps:

Algorithm 1:

a=a+1

Algorithm 2:

For i= 1 to n do:

a=a+1

end loop

Algorithm 3:

For i=1 to n do

For j= 1to n do

a=a+1

end loop

end loop

Now we do the analysis of these three algorithms and can see their performance with Big – O notation. In the algorithm 1 we may find that the execution statement *a=a+1* is the independent and is not constrained within any loop. Therefore, the number of times this will execute is 1. Thus, the frequency count of this algorithm is 1. Hence its **Time Complexity** is *O(1)*.

In the second algorithm, the execution statement *a=a+1* is inside the loop. The number of times it is executed is *n* as the loop runs for *n* times. The frequency count for this algorithm is *n*. Hence its **Time Complexity** is *O(n)*.

In the third algorithm, the frequency count for the execution statement *a=a+1* is n^2 as the inner loop runs *n* times, each time the outer loop runs, the outer loop also runs for n times. Hence its **Time Complexity** is $O(n^2)$.

Therefore during the analysis of algorithm we have the concern to determine the order of magnitude of an algorithm. Thus, we consider only those statements which may have the greatest frequency count.

2.6 COMMON COMPUTING TIMES OF ALGORITHM

The common computing times of algorithms in the order of their performance are as follows:

- *O(1)*: It means that the computing time of the algorithm is constant
- *O(log n)*: It means that the computing time of the algorithm is logarithmic
- *O(n)*: It means that the computing time of the algorithm is directly proportional to n. It is known as the linear time.
- *O(n log n)*: It means that the computing time of the algorithm is logarithmic
- $O(n^2)$: It is known as the quadratic time
- $O(n^3)$: It is known as the cubic time
- $O(2^n)$: It is known as the exponential time. Generally the algorithm with exponential time has no practical use.

There are different types of time complexities which can analyse for an algorithm:

- Best case time complexity: The best case complexity of an algorithm is a measure of the minimum time that the algorithm will require for a n input of size *'n'*. The running time of many

algorithm varies not only for the input of different size but for the different inputs of the same size.

- Average case time complexity: The time that an algorithm will require to execute input data of size *'n'* is known as average case time complexity. We can say that the value that is obtained by averaging the running time of an algorithm for all possible inputs of size *'n'* can determine average-case time complexity.
- Worst case time complexity: The worst time complexity of an algorithm is a measure of the maximum time that the algorithm will require for an input of size *'n'*. The worst case complexity is useful for a number of reasons. After knowing the worst case time complexity, we can guarantee that the algorithm will never take more than this time.

Hence the computation of exact time taken by the algorithm for its execution is very difficult. Thus, the work done by an algorithm for the execution of the input of size *'n'* defines the time analysis as function *f(n)* of the input data items.

2.7 EXAMPLE AND ANALYSIS

Example: This example exhibits the analysis of linear search algorithm complexity.

Consider the algorithm to search vector (array) V of size N for the location containing value X.

Algorithm SEARCH [Given a vector V containing N elements, this algorithm searches V for the value of a given X. FOUND is a Boolean variable. I and LOCATION are integer variables.]

1. [Search for the location of value X in vector V]

```
FOUND = false
    I=1
    Do while ((I<=N) && (FOUND ==false))
   If V[I] ==X
   Then FOUND==true
     LOCATION = I
    EXIT
  Else I = I + 1
  Print("Value of", X,"NOT FOUND")
```

2. [Finished]

```
Exit
```

Analysis : A reasonable active operation in the algorithm is the comparison between values of *V* and *X*. However, a problem arises in counting the number of active operations executed and the answer depends on the index of the location containing *X*. The **best case** is when *X* is equal to *V [1]* since only one comparison is used. The **worst case** is when *X* is equal to *V [N]* and *N* comparisons are used. Now to obtain the time of execution for the average case we need to know the probability distribution for the value *X* in the vector, i.e. the probability of X occurring in each location. If we assume the vector is not sorted, it is reasonable to assume that *X* is equally likely to be in each of the locations. But *X* might not be in the list at all. Let *q* be the probability that *X* is in the list. Then using the above assumption, we have;

Probability *X* is in location = *q/N;*

Probability *X* is not in the vector = *1-q;*

The average time is given by:

$T(avg) = \sum_{\text{sin } S}$ *(Probability of situation s) * (time for situation s)* [where *S* is the set of all possible situations where the *X* can be found]

$$T(avg) = \sum_{s=1}^{N} \frac{q}{N} * s + (1-q) * N = \frac{q}{N} \sum_{s=1}^{N} s + (1-q) * N = q * \frac{(N+1)}{2} + (1-q) * N$$

Thus if *q=1*, then: $T(avg) = \frac{(N+1)}{2}$

And if *q=1/2*, then: $T(avg) = \frac{(N+1)}{4} + \frac{N}{2} \approx \frac{3N}{4}$

So in either case the time is proportional to N.

Thus we obtain the time complexity for three cases as:

Best - case time for the linear search is *O (1)*

Worst-case time for the linear search is *O (N)*

Average-case time for the linear search is *O (N)*

Space Complexity

The space needed by the program is the sum of the following components:

- **Fixed space requirement:** This includes the instruction space, for simple variables, fixed size structured variables and constants.
- **Variable space requirement:** This consists of space needed by structured variables whose size depends on particular instance of variables.

2.8 SUMMARY

This unit described the algorithm and its analysis in very concise manner. The algorithm provides the way for solving the given problem in a systematic way. The following points are described in this unit:

- The term algorithm refers to the sequence of instructions that must be followed to solve a problem.
- An algorithm is a logical representation of the instructions which should be executed to perform a meaningful task.
- Analysis of the algorithm is done after determining the running time of an algorithm based on the number of basic operations it performs.
- There are two basic approaches for designing the algorithm i.e. Top-down approach and Bottom –Up approach.
- The running time varies depending upon the order in which input data is supplied to it.
- Analysis of an algorithm is done on the following basis:
    * Best case time complexity
    * Worst case time complexity
    * Average case time complexity
- Comparison of algorithm is done on the basis of the programming efforts for a program and on the basis of time and space requirements for the program.
- Big 'O' notation is extremely useful for classifying algorithms by their performances.
- Examples and analysis for computing the time complexity of the algorithm is explained.

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SELF EVALUATION

1. Algorithm must be:
   a. Efficient
   b. Concise and compact
   c. Free of ambiguity
   d. None of these
2. In top-down approach:
   a. A problem is subdivided into sub problems.
   b. A problem is tackled from beginning to end in one go.
   c. Sub-problems are solved first; these all solutions to sub-problems are put to solve the main problem.
   d. None of these.
3. Which one of the following is better computing time?
   a. O (N) b. $O(2^N)$
   c. $O(\log_2 N)$ d. None of the above
4. Define algorithm and design an algorithm to find out the total number of even and odd numbers in a list of 100 numbers.
5. Explain different ways of analyzing algorithm.
6. What is time and space complexity for the algorithm?
7. What is Big-O method for algorithm analysis?
8. Determine the complexity of the algorithm with Big - O notation for the following statement:

   for i = 1 to n

   for j = 1 to n

   for k = i to n

   *a = a*2;*

   b = b+1

   end loop

   end loop

   end loop

UNIT-3 ARRAY

Structure

3.0 INTRODUCTION

This unit discusses about one of the linear type data structure i.e. Array. It also presents about the various operations that can be performed on Arrays. This unit also presents about the two-dimensional and multidimensional arrays and their representations in row-major and column-major order. It also considers about the formulation of address calculation for single, two and multi dimensional arrays. In the last it introduces the concept of sparse matrices.

3.1 OBJECTIVE

Array is a linear data structure with contiguous memory location. The array data structure is used to store the same type of data type in sequential manner. At the end of this unit, you will be able to;

1. Understand the definition and representation of one dimensional array.
2. Know about the representation of two dimensional and multidimensional arrays in row-major and column-major way.
3. Calculation of address for one, two and multidimensional arrays.
4. Understanding and representation of the sparse matrices.

3.2 DEFINITION OF ARRAY

The simplest type of linear data structure is an **array**. The array is preferred for the situation which requires similar type of data items to be stored together as in contiguous memory location in static memory allocation manner. Thus, an array is a finite collection of similar elements stored in adjacent memory locations, for example an array may contain all integers or all characters. Therefore an array is a collection of variables of the same type that are referred by a common name. Alternatively we can also say that an array is a list of a finite number *n* of similar data elements referenced respectively by a set of *n* consecutive numbers, usually *1, 2, 3, …n.*

An array with *n* number of elements is referenced using an index that ranges from *0* to *n-1*. The lowest index of an array is called its **lower bound** and highest index is called the **upper bound**. The number of elements in an array is called its ***range or length or size*** of the array. For example the element of an array ***A[n]*** containing *n* elements are referenced as ***A[0], A[1], A[2],.....A[n-1]*** here the 0 (zero) is the lowest bound and *n-1* is the upper bound of the array. The array defined in this form is considered as the **linear array** or the **one-dimensional array**. The linear or one-dimensional array may also be defined as:

A ***linear array*** is a list of a finite number *n* of inhomogeneous data elements (i.e., data elements of the same type) such that:

a) The elements of the array are referenced respectively by an index set consisting of *n* consecutive numbers.

b) The elements of the array are stored respectively in successive memory locations.

As an example if we choose the name *A* for the array, then the elements of A are denoted by subscript notation: $a_1, a_2, a_3, \ldots\ldots, a_n$

Or, by the parenthesis notation: *A (1), A (2), A (3) ,……., A(N)*

Or, by the bracket notation: *A [1], A [2], A [3] ,……..., A[N]*

Regardless of the notation, the number *K* in *A [K]* is called a *subscript* and *A [K]* is called a ***subscripted variable.*** The general equation to find the length or the number of data elements of the array can define as:

Length = UB – LB + 1

Here, ***UB*** is the upper bound or largest index of the array and ***LB*** is the lower bound or the smallest index of the array. This is quite obvious that if: ***LB = 1*** then ***Length = UB***.

3.3 DECLARATION & INITIALIZATION OF ARRAY

Here we consider the declaration of array and its initialization. As per the majority of people are aware from the C language so we choose the syntax of C language for the declaration of array as well as for its initialization. Therefore an array can be declared just like any other variable in C i.e. data type followed by array name with subscription in bracket which indicates the number of elements it will hold. Thus by declaring an array, the specified number of memory locations i.e. the size of array are allocated in the memory. The declaration of array is specified with given example as:

int A[10]; // This is an integer type array where each element holds an integer value and in total 10 integers can hold.

float b_charge [20]; // This is a float type array where each element holds a float value and in total 20 values can hold.

char name [50]; // This is a character type array where each element holds a character value and in total 50 characters can hold.

The elements of an array can be easily processed as they are stored in contiguous memory locations and it can be seen from the following example:

int a [5]

This is stored as:

100	102	104	106	108
a[0]	a[1]	a[2]	a[3]	a[4]

Initialization of Array

Any array can be initialized at the time of its declaration as specified in following example:

int A[5] = [67, 102, 6, 8, 90];

float B[3] = [20.67, 100.78, 1000];

This array declaration and its initialization can be represented as:

	[0]	[1]	[2]	[3]	[4]
	67	102	6	8	90
Address⟼	100	102	104	106	108

In this array representation the address is mentioning the memory location i.e. the array A is of type integer and every integer occupies 2 bytes in the memory. So if the first element of array stores at memory location 100

then the next element of array will store on location 102 and so on. Thus, it is showing the contiguous memory location for the array.

Now we consider the example of an array of character and its representation in the form of contiguous memory location.

Char name [4] = "Ram"

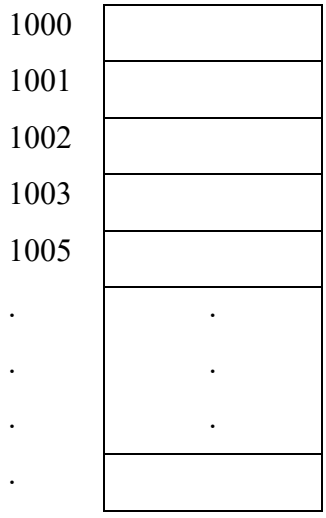
The values assigned in the *name* array as follows:

	[0]	[1]	[2]	[3]
	R	a	m	'\0'
Address⟼	100	101	102	103

An array of characters is called a ***string*** and it is terminated by a null character *('\0')*.

3.4 ONE DIMENSIONAL ARRAY AND ITS MEMORY REPRESENTATION

The one dimensional array is the simplest form of a linear array. The array is represented with its name and the elements those are referred by the subscripts or indices of the array. A one dimensional array is used to store a large number of items in memory. It references all item in uniform manner. Now we consider a linear array ***A*** in memory of the computer. As we know that the memory of the computer is simply a sequence of addressed location as:



Computer Memory with address location

Let us use the following notation when calculating the address of any element in linear array or one dimensional array:

LOC (A [k]) = address of the element *A [K]* of the array *A*. As we have discussed previously that the elements of any linear array stores in contiguous or successive memory locations. Therefore the computer does not need to keep track of the address of every memory element of the array but it needs to keep track of the address of the first element of the array. This address of the location of first element of the array we represent it by ***base address*** of the array as: ***Base (A)***. The address of any element of the array can be calculated by using this **base address** of the array as:

Loc (A [k]) = Base (A) + w (k- lower bound); here *w* is the number of words per memory cell for the array ***A.***

Note: The time to calculate *Loc (A [k])* is essentially the same for any value of *k*. We can locate and access the content of *A [K]* without scanning any other element of ***A***.

Examples of Array :

1. Let us consider an Array ***D*** of 5 – element linear array of integers such that:

D [1] = 247, D [2] = 56, D [3] = 429, D [4] = 135, D [5] = 87

The array ***D*** will be represented as:

247	56	429	135	87

2. Let us consider a company which uses an array ***C*** to record the number of items sold each year from 1932 to 1984.

Therefore rather than starting from the index 1 of the array we begin the index set with 1932. So we know that:

C [k] = number of items sold in the year k.

Then, *Lower bound* ***(LB)*** *= 1932* and the *upper bound* ***(UB)*** *= 1984* of the array *C*. Now we can find the length of the array as:

Length = UB – LB + 1 = 1984-1932+1 = 53.

Hence there will be the *53* elements in the array ***C*** and the index of the array will start from the index 1932 and ends on 1984.

3. Now from the example 2 of the array ***C*** if the base address of the array is 200 and w = 4 words per memory cell for array ***C.*** Now the base addresses of the following arrays are:

*Loc(**C [1932]**) = 200, Loc(**C [1933]**) = 204, Loc(**C [1934]**) = 208,* ……... Now we find the address of the array element for the year *k = 1965*. So that we have:

*Loc(**C [1965]**) = Base(**C**) + **w (1965 – LB) = 200 + 4 (1965 – 1932) = 332***

3.5 OPERATION ON LINEAR ARRAY

Let ***A*** be a collection of data elements stored in the memory of computer in successive memory locations. Now to print the contents of each element of array ***A*** or count the number of elements of ***A*** with a given property so that each element of ***A*** will have to be accessed or processed at least once is known as the **Traversing** of the array.

Algorithm Traversing :

[Let ***A*** be the linear array with lower bound ***LB*** and upper bound ***UB***. This algorithm traverses ***A*** for each element of ***A.***]

1. [Initialize an integer variable counter with value of lower bound]

   k = LB

2. [Repeat the following steps]

   Do while k < UB

   [read the elements of array into a temporary variable *temp* and print the read value]

   temp = A [k];

   print(temp p);

   [Increase the counter variable]

   k = k + 1

3. [Exit the loop]

   Exit.

Insertion and Deletion operation in Array

Let **A** be a collection of data elements in the memory of the computer. **"Inserting"** refers to the operation of adding another element to the collection **A**. Inserting an element at the end of a linear array can easily be done if memory space allocated to the array is large enough to accommodate the additional element. The element can also be inserted in the middle of the array. In this, on the average, half of the elements must be moved downward to new locations to accommodate the new element and keep the order of the other elements. The **"deleting"** refers to the operation of removing one of the elements from A. Deleting an element at the "end" of an array presents no difficulties, but deleting an element somewhere in the middle of the array would require that each subsequent element be moved one location upward in order to fill up the array.

Algorithm Insertion :

[Let **A** be a linear array. The function is **INSERT (A, N, K, ITEM)**. N is the number of items, K is the positive integer such that K <= N . The following algorithm inserts an element ITEM into the K^{th} position of array A.]

1. [Initialize Counter]

   J=N;

2. [Repeat step 2 and 3 till J>=K]

   While J>=K

   {

   A [J+1]= A [J]; /* Move J^{th} element downward */

   J=J-1; */* Decrease Counter */*

   }

3. [Insert element]

   A[K] = ITEM;

4. [Reset N]

   N= N+1;

5. *Exit*

Algorithm Deletion :

[Let **A** be a linear array. The function used to delete from the array is **DELETE (A, N, K, ITEM)**. N is the number of items, K is the positive integer such that K<= N. The following algorithm delete K^{th} element from the array.]

1. [Set the value of ITEM]

   ITEM = A[K]

2. [Repeat the step]

   for(J=K; J<=N-1;J++)

   A[J] = A[J+1] /* Move J+1 element upward */

3. [Reset the number N of elements in A]

   N=N+1

4. *Exit*

Example :

Consider **T** has been declared as a 5-element array but data have been recorded only for **T** [1], **T** [2], and **T** [3]. If X is the value to the next element, then we may simply assign, **T** [4] = X to add X to the Linear Array. Similarly, if Y is the value of the subsequent element, then we may assign, **T** [5] = Y to add Y to the Linear Array. Thus, we cannot add any new element to this Linear Array for **T** [6] because it exceeds the limit of this array upper bound.

3.6 TWO-DIMENSIONAL ARRAY

A two-dimensional array of size *m* x *n* is a collection of elements placed in *m* rows and *n* columns. Each element in array is specified by a pair of integers (such that j, k) knows as ***subscripts,*** with the following property:

$0 <= j <= m$ and $0 <= k <= n$

Thus, there are two subscripts in the syntax of 2-D array in which one specifies the number of rows and the other the number of columns. In a 2-D array each element is itself an array. Let **A** be a 2-D array. The element of **A** with first subscripts *j* and second subscript *k* is represented as:

$A_{j,k}$ Or $A[j,k]$

Any 2-D array is also called as the *matrix* in mathematics and *Table* in business application. Thus, a 2-D array is also called the *matrix arrays.*

An example of 2-D array can be **A[2][4]** containing 2 rows and 4 columns and **A[0][7]** is an element placed at 0^{th} row and 7^{th} column in the array. A 2-D array can be represented as:

	Column 0	Column 1	Column 2
Row 0	**A[0][0]**	**A[0][1]**	**A[0][2]**
Row 1	**A[1][0]**	**A[1][1]**	**A[1][2]**
Row 2	**A[2][0]**	**A[2][1]**	**A[2][2]**

Representation of 2-D array in memory

Example :

Let each student in a class of 10 students is given 4 tests. Assume the students are numbered from 1 to 10, the test scores can be assigned to a 10 x 4 matrix array SCORE as follows:

Student	Test 1	Test2	Test3	Test4
1	23	56	29	38
2	56	67	92	83
3	47	78	39	48
4	78	87	93	84
5	82	77	49	58
6	56	65	94	85
7	30	56	59	68
8	65	45	95	86
9	78	54	69	78
10	36	35	96	87

Representation of Array *SCORE*

Thus, ***SCORE [K, L]*** contains the K^{th} student's score on the L^{th} test. Hence the second row of an array is containing the four test scores of the second student.

SCORE [2, 1]=56 SCORE [2, 2]=67 SCORE[2,3] = 92 SCORE [2, 4]= 83

Let ***A*** is a 2-D *m* x *n* array. The first dimension of ***A*** contains the *index set* 1,……..,*m* with **lower bound** *1*and **upper bound** *m*. The second dimension of ***A*** contains the *index set 1,2,…..n,* with **lower bound** *1* and *upper bound n*. The length of a dimension is the number of integers in its index set. The product of length *m* x *n* is called the *size* of the array. Let us find the length of a given dimension i.e. the number of integers in its index set from the formula:

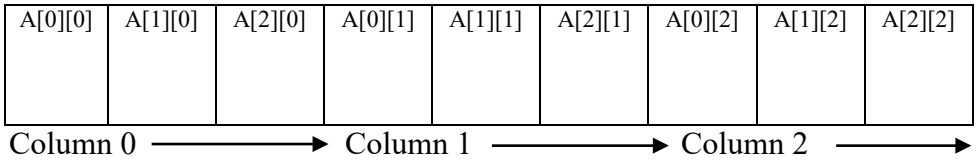
Length = upper bound – lower bound + 1

3.7 2-D ARRAY REPRESENTATION

As we know that all elements of a matrix or 2-D array set is stored in the memory in a linear fashion. Let ***A*** be a 2-D *m* x *n* array. Since ***A*** is pictured as a rectangular array of block of *m, n* sequential memory locations. This sequential memory representation can be considered in two ways; **Row Major Order** and **Column major Order.** In **row-major** representation the first row of the array occupies the first set of memory locations, second occupies the next set and so on. This form of the representation can be represented as:

Representation of 2-D Array in Row-Major Order

In **column-major** representation the first column of the array occupies the first set of memory location, second occupies the next and so on. This form of the representation can represent as:



Representation of 2-D Array in Column-Major Order

As we have discussed already about the linear array and its base address. A linear array ***A*** does not keep track of the address of every element of the array ***A,*** but does keep track of *Base* ***(A)***. Hence the address of the K^{th} element of ***A*** can be computed as:

$LOC(\mathbf{A}[K]) = Base(\mathbf{A}) + w*(K-1)$. Here w is the number of words per memory cell for the array ***A***, and 1 is the lower bound of the index set of ***A***.

3.8 ADDRESS IN 2-D ARRAY

A similar situation as we have discussed for the linear array also holds for any 2-dimensional m x n array ***A***. Hence the computer keeps track of *Base* ***(A)*** i.e. the address of the first element of ***A [0, 0]*** of ***A***. The address of any element say ***A[J, K]*** can compute for **row-major order** and also for **column- major order.**

(i) Row-major order:

The formula for *row-major order* is:

$LOC(\mathbf{A}[J,K]) = Base(\mathbf{A}) + w*[N(J-l)+(K-l)]$

Here w denotes the number of words per memory location for array ***A,*** l is the lower bound and N are the number of columns in the array.

(ii) Column-major order:

The formula for *Column-major order* is:

$LOC(\mathbf{A}[J,K]) = Base(\mathbf{A}) + w*[M(K-l)+(J-l)]$

Here w d enotes t he nu mber of words p er memory lo cation f or array ***A,*** *l* is the lower bound and M are the number of Rows in the array.

Example :

Calculate the address of an element in the following 2-D array:

int ***M*** [3][4] = { {1, 2, 3, 4}, {5, 6, 7, 8},{9, 10, 11, 12}};

The ba se a ddress of t he a rray ***M*** is 100. S ince *w = 2* (The ar ray i s o f integer type and integer occupies 2 byte in memory), according to the row-major formula the address of $(2, 3)^{th}$ element in the array ***M*** is:

LOC (2, 3) = 100 + 2[4(2-1) + (3-1)] = 100 + (4+2)*2 = 112*
[lower bound of the array is assumed to be 1]

Now a ccording t o t he column-major f ormula the a ddress of $(2, 3)^{th}$ element in the array ***M*** is:

LOC (2, 3) = 100 + 2[3(3-1) + (2-1)] = 100 + (6+1)*2=114*

3.9 MULTI-DIMENSIONAL ARRAYS

The arrays can also have more than two dimensions. For example, a three – dimensional (3-D) array may be declared as:

*int **A**[2][3][4];*

The number of elements in any array is the product of the ranges of all its dimensions. Therefore the array ***A*** contains 2*3*4 = 24 e lements of type integers. An element of this array is referenced with three subscripts. The first s pecifies th e p lane o r *rack* number, t he s econd s pecifies t he r ow number and the third specifies the column number. It is just like the library of books. The book is available in a particular column of the selected row in the specific rack. This array can be represented in memory as:

A[0][0][0]	*A[0][0][1]*	*A[0][0][2]*	*A[0][0][3]*
A[0][1][0]	*A[0][1][1]*	*A[0][1][2]*	*A[0][1][3]*
A[0][2][0]	*A[0][2][1]*	*A[0][2][2]*	*A[0][2][3]*
A[1][0][0]	*A[1][0][1]*	*A[1][0][2]*	*A[1][0][3]*
A[1][1][0]	*A[1][1][1]*	*A[1][1][2]*	*A[1][1][3]*
A[1][2][0]	*A[1][2][1]*	*A[1][2][2]*	*A[1][2][3]*

3-Dimensional Memory Representation if array in row major order

Similarly t he c olumn major or der can b e c onsidered. T he general multidimensional arrays are defined analogously. More specifically, an n-dimensional m_1 x m_2 x……x m_n, a rray ***A*** is a c ollection o f m_1, m_2, ……, m_n, data elements in which each element is specified by a list of

n integers such as $K_1, K_2, \ldots\ldots\ldots K_n$ called *subscripts,* with the property that:

$$\mathbf{0 <= K_1 <= m_1,} \quad \mathbf{0 <= K_2 <= m_2,} \quad \mathbf{\ldots\ldots\ldots} \quad \mathbf{0 <= K_n <= m_n,}$$

The element of B with subscripts $\mathbf{K_1, K_2, \ldots K_n}$ will be denoted by:

$$\mathbf{B[K_1, K_2, \ldots K_n]}$$

The array will be stored in memory in a sequence of memory locations. Specifically, the programming language will store the array B either in *row-major order* or *column-major order*.

3.10 SPARSE MATRICES

Any matrices with relatively high proportion of zero or null entries are called sparse matrices. Thus, a sparse matrix can be defined as a matrix with maximum number of zero entries. In the sparse matrix space and computing time could be saved if the non-zero entries were stored explicitly i.e. ignoring the zero entries the processing time and space can be minimized in sparse matrices. A sparse matrix can be divided into two categories:

- N^2 Sparse matrix: N^2 sparse matrix is a matrix with zero entries that form a square or a bar.
- Triangular sparse matrix: In this sparse matrix the zero entries are in its diagonal, either in the upper or lower side.

Now we consider the following sparse matrix:

$$\begin{vmatrix} 0 & 0 & 0 & 24 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 9 & 0 & 0 \\ 0 & 0 & 0 & 0 & 18 & 0 & 0 \\ 0 & 0 & 0 & 0 & 8 & 0 & 0 \end{vmatrix}$$

In this sparse matrix we have the 6 rows and 7 columns. There are 5 nonzero entries out of 42 entries. Therefore it requires an alternate form to represent the matrix without considering the null entries. Now we consider a data structure ***triplet*** to represent a sparse matrix. The ***triplet*** is a two dimensional array having t+1 rows and 3 columns. Here t is the total number of nonzero entries.

In this representation of ***triplet*** the first row contains number of rows, columns and nonzero entries available in the matrix in its 1st, 2nd and 3rd column respectively. Second row onwards it contains the row subscript, column subscript and the value of the nonzero entry in its 1st, 2nd and 3rd

column respectively. Now we represent the given sparse matrix in the ***triplet*** form of a 6 x 3 two dimensional array as:

6	7	5
1	4	24
2	6	5
4	5	9
5	5	18
6	5	8

Triplet representation of the given sparse matrix

There is another method also available for the representation of the sparse matrix. This method is known as **3-Tuple** method. In this method only the non-zero entries from the given sparse matrix are stored in three *tuples* form. These three *tuples* are: *row, column and value*.

Let us consider a sparse matrix with 3 rows and 4 columns as:

	Column 1	Column 2	Column 3	Column 4
Row 1	15	0	0	21
Row 2	22	11	0	0
Row 3	0	19	35	16

Now the 3-*tuple* representation of above matrix will be represented as:

	Row	Column	Value
A[0]	1	1	15
A[1]	1	4	21
A[2]	2	1	22
A[3]	2	2	11
A[4]	3	2	19
A[5]	3	3	35
A[6]	3	4	16

Example:

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Consider the following sparse matrix and represent it using array.

$$A = \begin{bmatrix} 15 & 0 & 0 & 21 \\ 22 & 11 & 0 & 0 \\ 0 & 19 & 35 & 16 \end{bmatrix}$$

As we know that as parse matrix is one where most of its elements are zero. The idea is to store information of non-zero elements. Information about non-zero elements has three parts:

- An integer representing its row.
- An integer representing its column.
- The data associated with its elements.

The elements of the above sparse matrix can be represented as follows using array:

0, 0, 15 0, 3, 21 1, 0, 22 1, 1, 11 2, 1, 19 2, 2, 35 2, 3, 16

3.11 SUMMARY

An array is a simplest type of linear data structure. The array is preferred for the situation which requires similar type of data items to be stored together as in contiguous memory location in static memory allocation manner. An array is a finite collection of similar elements stored in adjacent memory locations. This unit has described the linear data structure array. The contents of this unit can be summarized as:

- An array is a finite collection of similar elements stored in adjacent memory locations.
- There are many operations which could be performed on arrays like insertion, deletion, searching, sorting and traversing.
- Arrays can be single-dimensional, two-dimensional or multi-dimensional.
- There are two ways of representing two-dimensional arrays in memory i.e. Row-major and column-major order.
- Multidimensional arrays have more than two dimensions.
- Any matrices with relatively high proportion of zero or null entries are called sparse matrices.
- In the sparse matrix space and computing time could be saved if the non-zero entries were stored explicitly i.e. ignoring the zero entries the processing time and space can be minimized in sparse matrices.

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SELF EVALUATION

Multiple Choice Questions :

1. Elements of an array are accessed by:
   a. Accessing function in built –in data structure.
   b. Mathematical Function.
   c. Index.
   d. None of these.
2. Array is a:
   a. Linear data structure
   b. Non-linear data structure
   c. Complex data structure
   d. None of the above
3. Row-major order in 2-Dimensional array refers to an arrangement where:
   a. All elements of a row are stored in memory in sequence followed by next row in sequence and so on.
   b. All elements of a row are stored in memory in sequence followed by next column in sequence and so on.
   c. All elements of a column are stored in memory in sequence followed by next column in sequence.
   d. None of the above.
4. An element of sparse matrix consists of integers…….
   a. Two
   b. Three
   c. Six
   d. Ten
5. A sparse matrix is one where most of its elements are:
   a. Even
   b. Prime
   c. Zero
   d. Odd
6. An array ***A*** is declared as: double ***A[2][4];*** The array ***A*** has:

a. 2 elements

b. 4elements

c. 8 elements

d. None of these.

Answer the following Questions:

1. Write a 'C' function to find out the maximum and second maximum number from an array of integers.
2. Write a 'C' function to compute the product of two sparse matrices, represented with two-dimensional arrays.
3. Calculate the address of an element ***M*** [2][3] and ***M*** [3][1] in the following 2-D array:

   int ***M*** [4][4] = { {10, 12, 23, 14}, {15, 16, 17, 18},{19, 1, 2, 3}, {5, 6, 9, 4}};

   The base address of the array ***M*** is 200.
4. The number ***K*** in ***A[K]*** is called a…… and ***A[K]*** is called a…….
5. In linear array, the "downward" refers to……
6. At Maximum, an array can be a Three dimensional Array (True / False)
7. In……., the elements of a rray are stored column wise.
8. Write an algorithm to add the two one dimensional arrays and stored the sum in third array.
9. Write an algorithm to read the array in reverse order i.e. from upper bound to lower bound.
10. Calculate the address of an element ***A[4]*** and ***A[2]*** in the following one dimensional array

    int ***A[6] = {3, 6, 8, 1, 90, 62}***; The base address of the array is 100.

Uttar Pradesh Rajarshi Tandon
Open University

Postgraduate Diploma in

Computer Application

PGDCA-107

Data Structure

BLOCK

2

Stack, Queue and Recursion

Curriculum Design Committee

Dr.P.P.Dubey — **Coordinator**
Director, School of Agri. Sciences,
UPRTOU, Prayagraj

Prof. U. N. Tiwari — **Member**
Dept. of Computer Science and Engg.,
Indian Inst. Of Information Science and Tech.,
Prayagraj

Prof. R.S. Yadav, — **Member**
Dept. of Computer Science and Engg.,
MNNIT, Allahabad, Prayagraj

Prof. P. K. Mishra — **Member**
Dept. of Computer Science,
Baranas Hindu University, Varanasi

Mr. Prateek Kesrwani — **Member Secretary**
Academic Consultant-Computer Science
School of Science, UPRTOU, Prayagraj

Course Design Committee

Prof. U. N. Tiwari — **Member**
Dept. of Computer Science and Engg.,
Indian Inst. Of Information Science and Tech.,
Prayagraj

Prof. R.S. Yadav, — **Member**
Dept. of Computer Science and Engg.,
MNNIT, Allahabad, Prayagraj

Prof. P. K. Mishra — **Member**
Dept. of Computer Science,
Baranas Hindu University, Varanasi

Faculty Members, School of Sciences

Dr. Ashutosh Gupta, Director, School of Science, UPRTOU, Prayagraj

Dr. Shruti, Asst. Prof., (Statistics), School of Science, UPRTOU, Prayagraj

Ms. Marisha Asst. Prof., (Computer Science), School of Science, UPRTOU, Prayagraj

Mr. Manoj K Balwant Asst. Prof., (Computer Science), School of Science, UPRTOU, Prayagraj

Dr. Dinesh K Gupta Academic Consultant (Chemistry), Scool of Science, UPRTOU, Prayagraj

Dr. A cademic C onsultant (Maths), School of S cience, UPRTOU, Prayagraj

Dr. Dharamveer Singh, Academic Consultant (Bio-Chemistry), School of Science, UPRTOU, Prayagraj

Dr. R . P . S ingh, A cademic C onsultant (Bio-Chemistry), School of Science, UPRTOU, Prayagraj

Dr. S usma C huhan, A cademic C onsultant (Botany), School of S cience, UPRTOU, Prayagraj

Dr. Deepa pathak, Academic Consultant (Chemistry), School of Science, UPRTOU, Prayagraj

Dr. A. K. Singh, A cademic C onsultant (Physics), School of S cience, UPRTOU, Prayagraj

Dr. S . S . T ripathi, A cademic C onsultant (Maths), School of S cience, UPRTOU, Prayagraj

Course Preparation Committee

Prof. Manu Pratap Singh, **Author**
Dept. of Computer Science
Dr. B. R. Ambedkar University, Agra-282002

Dr. Ashutosh Gupta **Editor**
Director, School of Sciences,
UPRTOU, Prayagraj

Prof. U. N. Tiwari **Member**
Dept. of Computer Science and Engg.,
Indian Inst. Of Information Science and Tech.,
Prayagraj

Prof. R.S. Yadav **Member**
Dept. of Computer Science and Engg.,
MNNIT, Allahabad, Prayagraj

Prof. P. K. Mishra **Member**
Dept. of Computer Science
Baranas Hindu University, Varanasi

Dr. Dinesh K Gupta, **SLM Coordinator**
Academic Consultant- Chemistry School of Science, UPRTOU, Prayagraj

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RIL-102

BLOCK INTRODUCTION

This block will cover the two important types of linear data structures i.e. stack and queue with an important problem solving technique i.e. Recursion. This block includes the formal definition of these data structures and the method of their implementations. The array is used for the representation of these linear data structures. Therefore these data structures are implemented in sequential and static manner. There are various ope rations like insertion and delete are discussed for these two data s tructures. T he a pplications i n t he c omputer a nd f or c omputation of these d ata s tructures are d iscussed. E nough number o f ex amples i s discussed to show the operations of stack and queue.

Recursion i s a n i mportant c oncept i n c omputer s cience s pecially f or solving the many problems of recursive nature. Any problem is considered as recursive nature if the certain step of the problem or the entire problem is repeating with different parameters each time of repetition. Thus, many algorithms c an be b est de scribed i n t erms of r ecursion. R ecursion i s a n important f acility in m any programming la nguages. T here are ma ny problems w hose a lgorithmic de scription i s be st described i n a r ecursive manner. T he r ecursive i mplementation of va rious pr oblems i s di scussed with examples.

UNIT-4 STACK

Structure

4.0 INTRODUCTION

This unit is introducing the concept of another linear data structure i.e. Stack. It provides the definition of stack, its representation in memory, implementation procedure and different common and important operations those can perform on the elements of stack. This unit also includes the method for evaluation of arithmetic expressions using stack. In the end it highlights about the multiple stack concept and the different applications of the stack.

4.1 OBJECTIVES

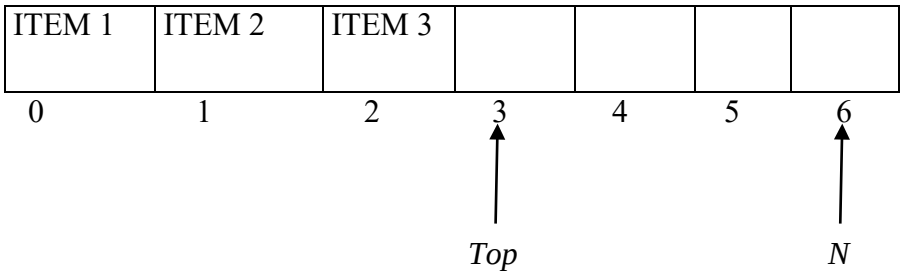
After going through this unit, you should be able to:

- Understand for the concept of stack
- Implementation of the stack using array.
- Implementation for the various operations on stack (Push, Pop).
- Understanding for the method of evaluation of arithmetic expressions using stack (infix, prefix and postfix representation).
- Understanding the concept of multiple stacks and its application.

4.2 STACK

A ***stack*** is a linear data structure where all the elements in the stack can insert and delete from one side only rather than at the middle or from both the side. Thus, from the stack the elements may be added or removed only from one side. The following figure shows the three everyday

stack. The *TOP* is pointing to 3 which says that stack has three items and as the *N* = 6, there is still space for accommodating four items.



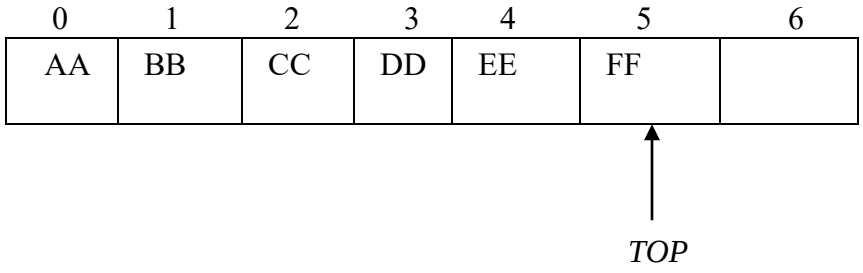
Array Representation of a Stack

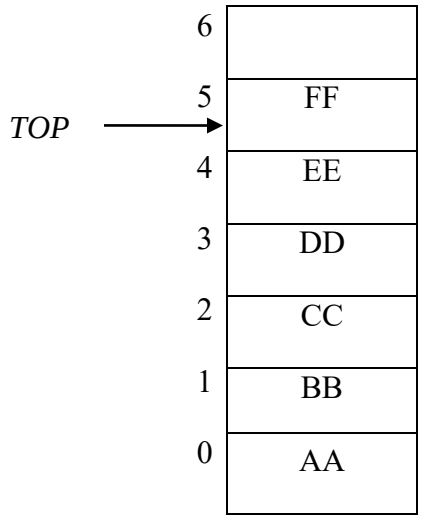
Example:

Suppose the following elements are inserted in order in an empty stack of size 7:

AA, BB, CC, DD, EE, FF

This stack can represent in following ways using a linear array:





4.4 OPERATION ON STACK

The operation to add an element into the stack (*push*) and the operation to remove an element from the stack (*pop*) can be implemented using the ***PUSH*** and ***POP*** functions. When we add a new element into the stack, first we check that whether there is a free space in the stack for the new element or not. If there is no free space available for the new element then we have the condition of overflow. In the same way for the function ***POP*** we must first check the condition that whether there is an element in the stack to be deleted or not. If there is no element in the stack to delete then we have the condition of *underflow*. These functions are defined as follows:

Function *PUSH (STACK, TOP, N, ITEM)*

[This function adds or pushes an ITEM in the stack]

```
/* check the condition of overflow*/
If (TOP == N)
Printf ("stack is overflow"); exit;
Else {
    TOP = TOP+1; /* Increase TOP by 1 */
    STACK [TOP] = ITEM; } /* Insert ITEM in new TOP position */
    RETURN
```

Function *POP (STACK, TOP, ITEM)*

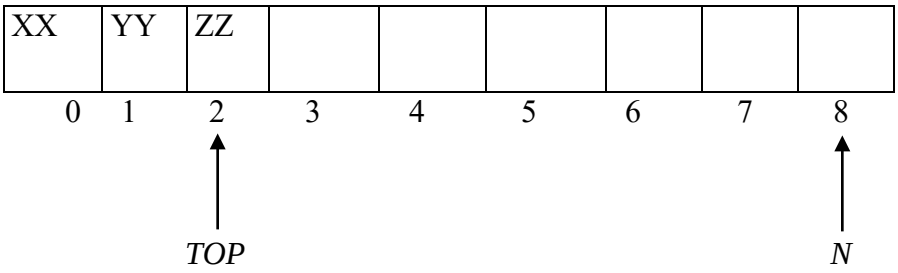
[This function deletes or pops the *TOP* element of *STACK* and assigns it to the variable *ITEM*]

```
/* check the condition of underflow*/
If (TOP == -1)
Printf ("stack is underflow"); exit;
Else {
    ITEM = STACK [TOP]; /* assign TOP element to ITEM */
    TOP = TOP – 1; } /* Decrease TOP by 1]
    RETURN
```

We can see from the above defined functions that the value of *TOP* is changed before adding the element in the stack in function ***PUSH*** but the value of *TOP* is changed after removing the element from stack in function ***POP***.

Example:

Consider the following stack with size of 9.

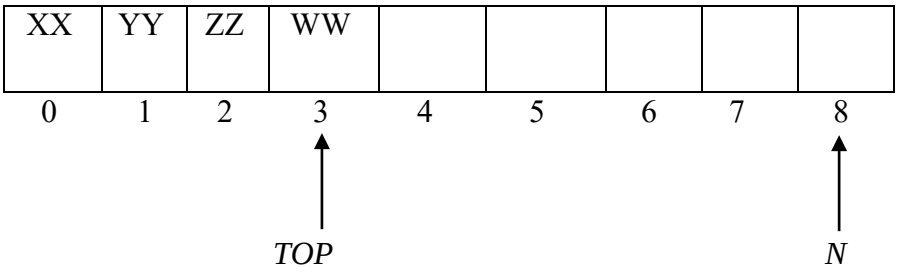


If we perform the operation ***PUSH (STACK, WW);*** then the status of the stack can determine as:

Since *TOP = 2*, so *TOP = 2 + 1 = 3*.

And *STACK [TOP] = STACK [3] = WW*.

Therefore the item *WW* is now top element of the stack. This can represent as:



Now, on the same stack we perform the operations ***POP (STACK, ITEM)***; ***POP (STACK, ITEM)***; then the status of the stack can determine as:

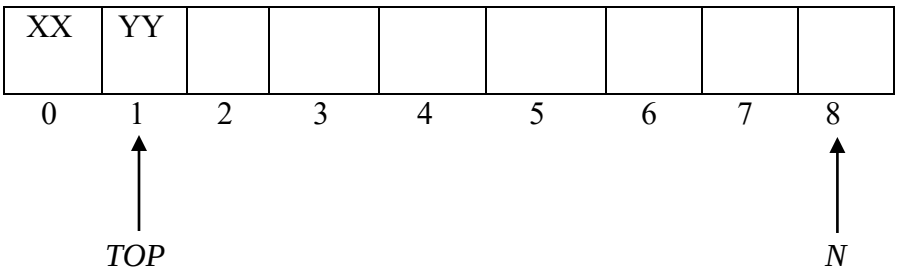
Right now the *TOP* = 3 and the operation *POP* is performed two times. So, after the execution of first *POP* operation:

ITEM = WW and *TOP = 3-1 = 2*

Now after the execution of second *POP* operation:

ITEM = ZZ and *TOP = 2-1 = 1*

Therefore *STACK [TOP] = STACK [1] = YY* is now the top element in the stack.



4.5 EVALUATION OF ARITHMETIC EXPRESSION (INFIX, POSTFIX AND PREFIX NOTIONS) USING STACK

This is well known that the computer system can understand and work only on binary paradigm. In which an arithmetic operation can take place between two operands only like *A + B, C * D, D / A*. generally an arithmetic expression may consist of more than one operator and two operands, for example *(A + B) * (D / (J + D))*. Such form of the arithmetic expression is commonly known as the *infix* expression. Normally the evaluation of the any arithmetic expression does not take place in its *infix* form. Here we are introducing the method for evaluating the *infix* expression form computation point of view. The ***stack*** is found to be more efficient to evaluate an infix arithmetical expression by first converting to a *prefix* or *postfix* expression and then evaluating these converted expressions. This approach will eliminate the repeated scanning of an infix expression in order to obtain its value. Therefore there are three basic notations are referred for the representation of any complex arithmetic expression. These forms are as follows:

- If the operator symbols are placed before its operands, then the expression is in ***prefix form***.
- If the operator symbols are placed after its operands, then the expression is in ***postfix form***.
- If the operator symbols are placed between the operands then the expression is in ***infix form***.

Hence, a normal arithmetic expression is normally called as infix expression i.e. A+B. A Polish mathematician found a way to represent the same expression called **polish notation** or prefix expression by keeping operators as prefix i.e. +AB. We use the reverse way of the above expression for our evaluation. The representation is called **Reverse Polish Notation** (RPN) or postfix expression i.e. AB+.

Let ***Q*** be an infix arithmetic expression involving constants and operations. This expression will be evaluated with the common rule of arithmetic evaluation with the following level of operator precedence:

Highest:	Exponentiation ($\uparrow$)
Next highest:	Multiplication (*) and division (/)
Lowest:	Addition (+) and Subtraction (-)

Now let we evaluate the following parenthesis free arithmetic expression:

$$2 \uparrow 3 + 5 * 2 \uparrow 2 - 12 / 6$$

First we evaluate the exponentiations to obtain:

*8 + 5 * 4 – 12 / 6*

Then we evaluate the multiplication and division to obtain *8 + 20 – 2*. Last, we evaluate the addition and subtraction to obtain the final result i.e. 26. We can observe that in this evaluation the whole expression is traversed three times, each time corresponding to a level of precedence of the operations.

Second important issue about the *infix notation* is that these expressions use parentheses for clarity and to make the evaluation convenient like (*A + (B – C)) * D) / E*. On the other hand the polish notation expression i.e. *prefix* and reverse polish notation expression i.e. *postfix* do not include any parentheses for clarity.

Now we consider for example the step by step translation of following infix expression into polish notation using square brackets i.e. [ ] to indicate a partial translation:

$$(A+B)*C = [+AB]*C = *+ABC$$
$$A+(B*C) = A+[*BC] = +A*BC$$
$$(A+B)/(C-D) = [+AB]/[-CD] = /+AB-CD$$

The fundamental property of Polish notation is that the order in which the operations are to be performed is completely determined by the positions of the operators and operands in the expression. There is no need of parentheses when writing expressions in Polish notation. The computer usually evaluates an arithmetic expression written in infix notation in following two steps.

- Coverts the expression in Reverse Polish notation form (*post fix notation).*
- Evaluate the *post fix* expression using stack.

Algorithm for transforming Infix Expression into Postfix Expression

Let ***Q*** be an arithmetic expression written in infix notation. The following algorithm transforms the given in fix expression ***Q*** into its equivalent postfix expression ***P***. This algorithm uses a stack to temporarily hold operators and left parentheses. The postfix expression ***P*** will be constructed from left to right using the operands from ***Q*** and the operators which are removed from STACK. We begin by pushing a left parenthesis onto STACK and adding a right parenthesis at the end of ***Q***. The algorithm is completed when STACK is empty:

Algorithm: Conv_POLISH (*Q, P)*

[Suppose ***Q*** is an arithmetic expression written in *infix* notation. The algorithm finds the equivalent *postfix* expression ***P***]

1. *Push "(" onto STACK, and add ")" to the end of* ***Q.***
2. *Scan* ***Q*** *from left to right and repeat step3 to 6 for each element of* ***Q*** *until the STACK is empty.*

3. *If an operand is encountered, add it to **P**.*

4. *If a left parenthesis is encountered, push it onto STACK.*

5. *If an operator is encountered, then:*

   *a. Repeatedly pop from STACK and add to **P** each operator (on the top of STACK) which has the same precedence as or higher precedence than operator.*

   b. Add operator to STACK.

   [end of If]

6. *If a right parenthesis is encountered, then:*

   *a. Repeatedly pop from STACK and add to **P** each operator (on top of STACK) until a left parenthesis is encountered.*

   *b. Remove the left parenthesis. [Do not add the left parenthesis to **P**.]*

   [End of if]

   [End of step 2 loop]

7. *Exit.*

Example:

Consider the following arithmetic *infix* expression:

***Q**:* *A + (B * C - (D / E ↑ F) * G) * H)*

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

The elements of ***Q*** have now been labeled from left to right for easy reference. Following table shows the status of STACK and of the Postfix string ***P*** as each element of ***Q*** is scanned to follow the following steps of algorithm.

1. Each element is simply added to ***P*** and does not change STACK from A to C; operators +. (, * are pushed to stack and A, B, C are added to P.

2. The subtraction operator (- in row 7 sends * from STACK to ***P*** before it (-) is pushed onto the STACK.

3. The right parenthesis in row 14 sends ↑ and then / from STACK to ***P*** and then removes the left parenthesis from the STACK.

4. The right parenthesis in row 20 sends * then + from STACK to ***P*** and then removes the left parenthesis from the top of STACK.

   After step 20 is executed, the stack is empty.

Symbol Scanned		STACK	Expression *P*
(1)	A	(	A
(2)	+	(+	A
(3)	(	(+(	A
(4)	B	(+(	AB
(5)	*	(+(*	AB
(6)	C	(+(*	ABC
(7)	-	(+(-	ABC*
(8)	(	(+(-(	ABC*
(9)	D	(+(-(	ABC*D
(10)	/	(+(-(/	ABC*D
(11)	E	(+(-(/	ABC*DE
(12)	↑	(+(-(/↑	ABC*DE
(13)	F	(+(-(/↑	ABC*DEF
(14)	)	(+(-	ABC*DEF↑/
(15)	*	(+(-*	ABC*DEF↑/
(16)	G	(+(-*	ABC*DEF↑/G
(17)	)	(+	ABC*DEF↑/G*-
(18)	*	(+*	ABC*DEF↑G*-
(19)	H	(+*	ABC*DEF↑G*-H
(20)	)		ABC*DEF↑G*-H*+

Evaluation of Post Fix Expression

As we know that in the *infix* expression it is difficult for the computer to keep track of precedence of operators. On the other hand, a *postfix* expression itself determines the precedence of operators due to the placement of the operator. Hence it is easier for the computer to perform the evaluation of a postfix expression. The evaluation rule for the *postfix* expression is stated as:

1. *Read the expression from left to right.*
2. *If it is an operand then push the element into the stack.*
3. *If the element is an operator except NOT operator, pop the two operands from the stack and evaluate them with the read operator and push back the result of the evaluation into the stack.*
4. *If it is the NOT operator then pop one operand from the stack and then evaluate it and push back the result of the evaluation into the stack.*
5. *Repeat it till the end of stack.*

Now we define the algorithm for evaluation of postfix expression using STACK. Let ***P*** be a arithmetic expression written in postfix notation. The following algorithm uses the STACK to hold the operands and evaluate expression ***P.***

[This algorithm finds VALUE of an arithmetic expression ***P*** written in *postfix* notation.]

1. *Add a right parenthesis ")" at the end of **P.***
2. *Scan **P** from left to right and repeat step 3 and 4 for each element until ")" is not encountered.*
3. *If an operand is encountered , put it on STACK*
4. *If an operator is encountered then:*
    a. *Remove the two top elements of STACK, where A is the top element and B is the next –to-top element.*
    b. *Evaluate B and A for the encountered operator.*
    c. *Place the result of evaluation on step b back on STACK*

    [End of if]

    [End of step 2 loop].
5. *Set VALUE equal to the top element on STACK.*
6. *Exit.*

Example

Evaluate the following arithmetic expression ***Q*** written in *infix* notation:

Q: *10 * (8 + 4) – 6 / 3*

The equivalent *postfix* notation for the given *infix* notation is:

P: *10, 8, 4, +, *, 6, 3, /, -* [Here Commas are used to separate the elements of ***P***]

Now we apply the algorithm to evaluate the *postfix notation:*

First we add ')' at the end of right side in expression ***P***.

P: 10, 8, 4, +, *, 6, 3, /, -,)

(1) (2) (3) (4) (5) (6) (7) (8) (9) (10)

The evaluation procedure with contents of STACK can consider from the following table:

Symbol Scanned	Stack
(1) 10	10
(2) 8	10, 8
(3) 4	10, 8, 4
(4) +	10, 12
(5) *	120
(6) 6	120, 6
(7) 3	120, 6, 3
(8) /	120,2
(9) –	118
(10))	

The final number in STACK is 118, which will assign to the VALUE when ')' encounters. Thus the evaluation of *postfix notation* ***P*** is 118.

Algorithm to convert *infix* into *prefix* expression form

Suppose ***Q*** is an arithmetic expression written in infix form. The following steps find its equivalent *prefix* expression ***P.***

1. *Push ')' onto STACK and add '('to the begin of* ***Q.***
2. *Scan* ***Q*** *from right to left and repeat steps 3 to 6 for each element of* ***P*** *until the STACK is empty.*
3. *If an operand is encountered add it to* ***P.***
4. *If a right parenthesis is encountered, push it onto stack.*

5. *If an operator is encountered then:*
   a. *Repeatedly pop from STACK and add to **P** each operator (on the top of STACK) which has same or higher precedence than the operator.*
   b. *Add operator to STACK.*
6. *If a left parenthesis is encountered then*
   a. *Repeatedly pop from the STACK and add to **P** until a right parenthesis is encountered.*
   b. *Remove the Right parenthesis*
7. *Exit*

Example:

Convert the following given *infix expression* in its equivalent *prefix notation form*.

Q: ***(A + B * C – D + E / (F + G))***

The procedure for converting the given *infix expression* into its equivalent *prefix notation* form by applying the algorithm can represent in following table:

Symbol Scanned	**Stack**	***Prefix* expression**
)		
)	))	
G	))	G
+	)) +	G
F	))+	FG
(	Empty	+FG
/	)/	+FG
E	)/	E+FG
+	)+	/ E+FG
D	)+	D/E+FG
-	)+-	D/E+FG
C	)+-	CD/E+FG
*	)+-*	CD/E+FG
B	)+-*	BCD/E+FG
+	)+-+	*BCD/E+FG
A	)+-+	A*BCD/E+FG
(	Empty	+-+A*BCD/E+FG

Hence the resultant equivalent *prefix notation* for the given *infix notation* is:

P: *+, -, *, +, A, B, C, D, / E + F G*

Algorithm for evaluation of Prefix Expression

[This algorithm performs the evaluation for the *infix notation* expression. Here the expression will read from right to left]

1. *Read the next element.*
2. *If element is operand then*
    a. *Push the element in the stack*
3. *If element is operator then*
    a. *Pop two operands from the stack*
    b. *Evaluate the expression formed by two operands and the operator*
    c. *Push the results of the expression in the stack*
4. *If no more elements then*
    a. *Pop the result*

Else

Go to step 1.

Example:

Evaluate the following *prefix notation* expression.

P: *+ 2 * 3 + 4 5*

Now we start to read it from right to left: *+2 * 3 + 45+*

Symbol Scanned	Stack
5	5 (push)
4	4, 5 (push)
+	9 (pop, push)
3	3, 9 (push)
*	27 (pop, push)
2	2, 27 (push)
+	29 (pop, push)

The final number in STACK is 29. Thus the evaluation of pre*fix notation* ***P*** is 29.

4.6 APPLICATION OF STACKS

The stack has various applications in computer science. It includes the application of the level of computer organization and programming level. Generally being a linear data structure the following applications of stacks are highlighted:

- The stack is used for reversal of a given list. We can accomplish this task by pushing each element onto the stack as it is read. When the line is finished, elements are then popped off the stack, so they come off in reverse order.
- The important application of the stack in computer organization is for the evaluation of arithmetic expressions. It accomplishes by converting first the given expression in reverse polish notation and then evaluates the expression.
- It is also used for the zero address instruction implementation in computer organization.
- Stacks are used for the address holding in function calling procedure of programming.
- The stacks are used to implement recursive procedures. Recursion is useful in developing algorithm for specific problems. Suppose a function contains either a call statement to itself or a call statement to a second function that may eventually result in a call statement back to the original function. Then such a function is called recursive function. The stacks are used generally for the implementation of such type of recursive functions.

4.7 SUMMARY

In this unit we presented another important linear data structure i.e. Stack. A stack is a linear data structure where all the elements in the stack can insert and delete from one side only rather than at the middle or from both the side. We have explored the implementation of stack in static manner using array. The two basic operations of PUSH and POP are discussed with example for the stack. The contents can be summarized as follows:

- A stack is a linear structure implemented in LIFO (Last In First Out) manner where insertions and deletions take place at the same end.
- An insertion is a stack is called pushing and deletion from a stack is called popping.

- When a stack implemented as an array, is full and no new element can be accommodated, it is called OVERFLOW.
- When a stack is empty and an attempt is made to delete an element from the stack, it is called UNDERFLOW.
- The main application of stack can be implementation of Polish notation which refers to a notation in which operator symbol is placed either before its operands (prefix notation) or after its operands (postfix notation). The usual form, in which operator is placed in between the operands, is called infix notation.
- The other application of stack can be reversing a list and providing recursion in various programs.
- It is also used for the zero address instruction implementation in computer organization.
- Stacks are used for the address holding in function calling procedure of programming.

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SELF EVALUATION

Multiple Choice Questions:

1. ..... form of access is used to add and remove nodes from a stack

   (a) LIFO (b) FIFO

   (c) Both (a) and (b) (d) None of these

2. A data structure in which elements are added and removed only at one end is know as:

   (a) Queue (b) Stack

   (c) Array (d) None of these

3. Underflow is a condition where you:

   (a) Insert a new node when there is no free space for it

   (b) Delete a non-existent node in the list

   (c) Delete a node from the empty list

   (d) None of the above

4. Stack is:

   (a) Static data structure

   (b) Dynamic data structure

   (c) A built in data structure

   (d) None of these

5. Which operation in the stack is used for getting value of most recent node and delete the node:

   (a) PUSH

   (b) POP

   (c) Empty

   (d) None of these

6. If A, B, C are inserted into a stack in the lexicographic order, the order of removal will be:

   (a) A, B, C

   (b) C, B, A

   (c) B, C, A

   (d) None of these.

Fill in the blank:

1. A stack may be represented by a …… linked list. (linear / non-linear)
2. Push operation in stack may result in…… (overflow / underflow)
3. If TOP points to the top of stack, then TOP is……. (increased / decreased)

State whether True or False

1. Push operation in stack is performed at the rear end.
2. PUSH operation in stack may result in underflow
3. For a stack implemented with linear array arbitrary amount of memory can be allocated.

Descriptive Questions

1. Consider the following stack, where ***STACK*** is allocated ***N = 6*** memory cells.

   STACK: ***AA, DD, EE, FF, GG……..***

   Describe the stack as the following operations take place and also consider the overflow condition.

   a. PUSH (***STACK, KK***)

   b. POP (***STACK, ITEM***)

   c. PUSH (***STACK, LL***)

   d. PUSH (***STACK, SS***)

   e. POP (***STACK, ITEM***)

   f. PUSH (***STACK, TT***)

2. Write an algorithm which upon user's choice, either pushes or Pops an element from the stack implemented as an array (the element should not shifted after the push or pop).

3. Write a program to convert an infix arithmetic expression into a prefix arithmetic expression. The algorithm for your program should use the following expression:

   *Q: (A – B) * (C / D) + E*

   Show in tabular form the changing status of stack.

4. Convert the expression *(A + B) / (C – D)* into postfix expression and then evaluate it for A = 10, B = 20, C = 15, D = 5. Display the stack status after each operation.

5. Write a program to read a string (one line of characters) and push any vowels in the string to a stack. Then pop your stack repeatedly and count the number of vowels in the string.

UNIT-5 RECURSION

Structure

5.0 INTRODUCTION

This unit introduces the concept of Recursion. It highlights the basic concepts of recursion, its definition with the method for its working. The solutions of some problems are performed with the recursive method. The unit also considers the recursive algorithm for solving the problems. It also explores the use of stack for recursive algorithms or finding the solution with recursive method.

5.1 OBJECTIVES

At the end of this unit, you may be able to:

- Understand the concept of Recursion and recursive method.
- Understand the principle of recursion and use of stack for the implementation of recursive methods.
- Understand the working of recursive procedures and implementation it for the solution of problems.
- Designing of Recursive algorithm for the solution of some popular problems.

5.2 DEFINITION OF RECURSION

Recursion is an important concept in computer science specially for solving the many problems of recursive nature. Any problem is considered as recursive nature if the certain step of the problem or the entire problem is repeating with different parameters each time of repetition. Thus, many algorithms can be be st described in terms of recursion. Suppose ***P*** is any function containing either a Call statement to itself or a call statement to a second function that may eventually result in a call statement back to the original function ***P***. The ***P*** is called a *recursive*

function. Thus, a **recursive function** is a function that either directly or indirectly makes a call to itself. The important aspect is that the function ***P*** calls itself on a different, generally simpler instance, for example files on a computer are generally stored in directories. Users may create subdirectories that store more files and directories. Suppose that we want to examine every file in a directory D, including all files in all subdirectories and subdirectories, and so on. We do so by recursively examining every file in each subdirectory and then examining all files in the directory D. Hence this logic seem to be circular logic but with the method to come out from this infinite circular loop.

Therefore, a recursion is an important facility in many programming languages. There are many problems whose algorithmic description is best described in a recursive manner. Hence, a function is called recursive if the function definition refers to itself or does refer to another function which in turn refers back to the same function but this procedure should not include infinite loop or endless process. Hence in order for the definition does not contain any endless circular process, t must have the following properties:

(i) There must be certain arguments called base values, for which the function does not refer to itself. Alternatively there must be certain criteria, called ***base criteria,*** for which the function does not call itself.

(ii) Each time the function does refer to itself, the argument of the function must be closer to the base value.

Any recursive function with these two properties is said to be ***well defined*** recursive function. Similarly, a function is said to be ***recursively defined*** if the function definition refers to itself.

5.3 PROCESS OF RECURSION

Recursion is a powerful problem-solving tool. Many algorithms are most easily expressed in a *recursive formulation*. Furthermore, the most efficient solution for many problems is based on this recursive formulation but this formulation does not contain any infinite loop and process. Generally the idea of Recursion is closely related to the principle of *mathematical induction* which provides the following issues for the recursion procedure:

- Solve the problem for small problem instances.
- Assume a solution for smaller problem instance.
- Figure out how to do a little more work which in combination with solutions to smaller instances, solves the larger problem instance.

It can see that sometimes mathematical functions are defined recursively. Let us consider the example of sum *S (N)* for the first *N* integers. The base condition is defined as S *(1) = 1*, and the recursive function can define as:

$$S(N) = S(N-1) + N$$

Here we have defined the function *S* in terms of a smaller instance of itself. The straight forward recursive evaluation of the sum of the first *N* integers based on the recursive function as given above with its base condition can define as:

[Recursive function to compute sum of first n integers.]

```
Function sum (int n)
{
If (n == 1)   /* base condition */
Return 1;
Else
Return (s (n – 1) + n);
}
```

We can realize from the above algorithm that if *N=1,* we have the basis, for which we know that *S (1) = 1* and the recursive step is with the return statement as *S (N) = S (N-1) + N*. Now from the algorithm we can get an idea for the working process of the recursive method and as well as for the recursive function. We can consider in the above algorithm that a base case is an instance that we can solve without recursion. Any recursive call must progress towards the case in order to terminate eventually. Thus we have the two fundamental **rules of recursion:**

1. *Base case :* Always have at least one case that can be solved without using recursion.
2. *Make Progress :* Any recursive call must progress toward a base case.

This is quite obvious that if the base condition is not available for the recursive function then the function will stuck in infinite recursive call and never terminates.

5.4 DESIGNING FOR RECURSIVE ALGORITHM

We have discussed about the recursive function and recursive process. The important point which we have analyzed is that any recursive function should have a *based criteria* or *base condition* or *termination condition* otherwise the recursive procedure becomes unsolvable. Thus in order to design any recursive process in terms of the algorithm, the base

condition and progress condition should clearly described and stated. For example the problem of *factorial* can be solved using a recursive procedure. Now we consider the factorial of number and its algorithm described recursively as:

The product of the positive integers from *1* to *n*, inclusive, is called "n factorial" and is usually denoted by *N!* :

N! = 1.2. 3.4.5.....(N-2)(N-1)N for every positive integer N

So that we have, N! = N * (N-1)!

(N-1)! = N-1 * (N-2)! And so on up to 1.

Therefore we can define the factorial function in the form of a recursive function with its two conditions as:

(a) If *n = 0,* then *n! = 1.*

(b) If *n>0,* then *n! = n * (n-1)!*

We define the function ***FACT*** which finds the factorial of the given number N from recursive process:

/* Function:*/ int FACT (N)

{

if N==0 return 1

else

if

N==1

return 1

else

*return (N * FACT(N-1))*

}

To perform this recursive algorithm, let us consider *N = 5*. Hence according to the definition we can see that the *FACT (5)* will call *FACT (4), FACT (4)* will call *FACT (3), FACT (3)* will call *FACT (2)*, and *FACT (2)* will call *FACT (1)*. The execution will return back by finishing the execution of *FACT (1)*, then *FACT (2)* and so on up to *FACT (5)* as described below:

1) *5! = 5 * 4!*

2) *4! = 4 * 3!*

3) *3! = 3 * 2!*

4) *2! = 2 * 1!*

5) *1! = 1*

6) *2! = 2 * 1 = 2*

7) *3! = 3 * 2 = 6*

8) *4! = 4 * 6 = 24*

9) *5! = 5 * 24 = 120*

From above it is clear that every sub function contain parameters and local variables. The parameters are the arguments which receive values from objects in the calling program and which transmit values back to the calling program. The sub-function must also keep track of the return address in the calling program. This return address is essential since control must be transferred back to its proper place in the calling program. After completion of the sub-function when the control is transferred back to its calling program, the local values and returning address is no longer needed. Suppose our sub-program is a recursive one, when it calls itself, then current values must be saved, since they will be used again when the program is reactivated. Thus, in recursive process a data structure is required to handle the data of ongoing called function and the function which is called at last must be processed first i.e. the data accessed last must be processed fist i.e. Last in first out principle. So, a ***stack*** may be suitable data structure that follows LIFO to implement recursion. Thus when a recursive method is executed, each invocation of the method gets a separate stack frame. Hence each invocation has a separate copy of the following:

- Formal parameters
- Local variables
- Return value

Therefore the recursion is useful in developing algorithms for specific problems and the **Stack** may use to implement recursive functions or processes. Now we discuss the method for translating a recursive method into a non-recursive method using stack.

5.5 EXAMPLES OF RECURSIVE ALGORITHMS

RIL-102

Here we are presenting some examples of very common and important recursive algorithms:

1. **Fibonacci number**: Here we are defining the recursive algorithm for computing the Nth Fibonacci number. As we know that the next Fibonacci number we can compute by adding the previous two numbers i.e. the *nth* Fibonacci number we can find as:

   Fib(n) = fib(n-1)+fib (n-2)

   Fib (n-1) = fib (n-2) + fib (n-3)

   And so on, in the last we can see *Fib(3) = fib (2) + fib (1)*

   Here fib (1) = 1 and fib (2) = 1.

Thus we can see that the process of computing the Fibonacci number is a recursive with two base conditions i.e. *fib (1) = 1 and fib (2) = 1.* Hence the recursive algorithm for computing the Fibonacci number can define as:

```
Int fib (int n)
{
        If (n <= 1)
        Return n;
        Else
        Return fib (n-1) + fib (n – 2);
}
```

2. **Greater common divisor of two integers**: Here we are defining the recursive algorithm for obtaining the greatest common divisor of two integers.

```
Int gcd (int a, int b)
{
        If (b == 0)
        Return a;
        Else
        Return gcd (b, a % b);
}
```

For example if we compute the gcd of 70 and 25 then the above recursive algorithm will execute as:

First the recursive function will call with *gcd (70, 25).* On next iteration it will call with *gcd(25,20).* After that the third call will call with *gcd(20,5).* The last call of the recursive function will be of *gcd(5,0)⟹5.* Here the base condition satisfied and the recursive procedure will terminate. Therefore the gcd of these two numbers is 5 as:

gcd (70, 25) ⟹gcd(25,20)⟹gcd(20,5)⟹gcd(5, 0)⟹5

3. **Tower of Hanoi:** The Tower of Hanoi is an example of a problem that is much easier to solve using recursion rather than non-recursive method. The problem is defined as follows:

   - There are 3 pegs and n disks, all of different sizes
   - Initially all disks are on the start peg, stacked in decreasing size, with largest on bottom and smallest on top.
   - We must move all the disks to the end peg, one at a time and without ever putting a larger disk on top of a smaller disk.
   - The third peg can be used as a spare.

Initially we explore the solution for the 2 disks i.e. *n = 2*.After that we consider the recursive algorithm for the generalize case i.e. for *n* disks.

1. Move smaller disk from start peg to spare peg.
2. Move larger disk from start peg to end peg.
3. Move smaller disk from spare peg to end peg.

Now we consider the recursive procedure for solving this problem with *n* disks as:

1. Move the top *n-1* disks from the start peg to the spare peg using recursive call
2. Move the bottom disk directly from the start peg to the end peg.
3. Move the n -1 disks from the spar peg to the end peg using recursive call.
4. The *n = 1* will work as the base condition for recursive process. Therefore the peg can be moved directly.

The algorithm of the whole process can describe as:

```
Function Tower (int n, int stast, int finish, int spare)
{
        If (n == 1)
        Move disk from start to finish
        Else
        {
        Tower (n-1, start, spare, finish) /* move n-1 disks from start to spare */
        Move disk from start to finish /* move bottom disk directly to finish */
```

Tower (n-1, spare, finish, start) / move n-1 disks from spare to finish */*

}

}

5.6 SUMMARY

Recursion is useful in developing algorithm for specific problems. Suppose a function contains either a call statement to its self or a call statement to a second function that may eventually result in a call statement back to the original function. Then such a function is called recursive function. The stacks are used generally for the implementation of such type of recursive functions. Any problem is considered as recursive nature if the certain step of the problem or the entire problem is repeating with different parameters each time of repetition. Thus, many algorithms can be best described in terms of recursion. The contents of this unit can be summarized as follows:

- Recursion is the name given to the phenomenon of defining a function in terms of itself.
- There must be base condition in the recursive definition of any process which indicates its initial condition.
- Each time the function does refer to its self, the argument of the function must be closer to the base value.
- Recursion is closely related to the principle of mathematical induction.
- A stack may be suitable data structure that follows LIFO to implement recursion.
- Important examples like factorial, GCD and tower of Hanoi are explained and implemented with the help of recursive process.

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SELF EVALUATION

1. State whether it is *TRUE* or *FALSE*: " Recursion is generally more efficient than iteration"
2. What are the two fundamental rules of recursion?
3. Stack is used whenever …….. function is called (Recursive / Non- recursive)
4. Write a program for evaluating $\binom{n}{r}$ for the given value of n and r using recursive procedure.
5. Formulate the recursive function for evaluating the least common multiplier (LCM).

UNIT-6 QUEUE

Structure

6.0 INTRODUCTION

This unit introduces the concept of another important linear data structure used to represent a linear list i.e. Queue. This unit starts by giving an introduction to the basic concept of queue. It also defines the operation of insertion and deletion from the Queue. The queue allows insertion of an element to be made at one end and deletion of an element to be performed at the other end. This unit further introduces various type of queues like circular queue, de-queue and priority queue. It also provides the operation of insertion and deletion in these types of queues. In the last it highlights the application of queue.

6.1 OBJECTIVES

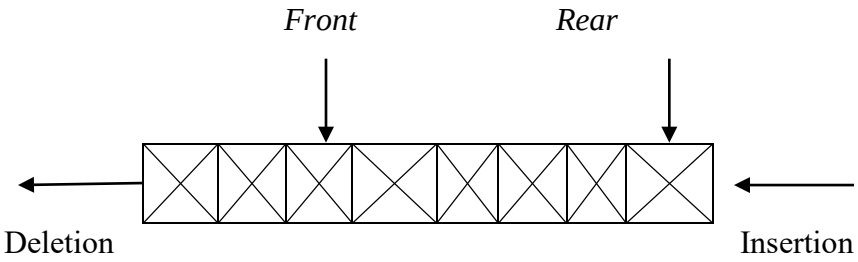
After working through this unit, you should be able to:

- Understand the concept of queue and its working with its definition.
- Implementation of queue using Array and perform the operation of insertion and deletion in it with the condition of overflow and underflow of the queue.
- Understand the concept of circular queue and its implementation with array.

- Implementation of insertion and deletion operations in the circular queue with overflow and underflow condition.
- Understand the concept of de-queues and priority queues with their implementation details using array.
- Familiarity with the different application of queues in computer.

6.2 DEFINITION OF QUEUE

Queue is linear data structure in which the element is inserted from one end of the queue called *rear*, and the deletion of the element from other end of the queue called *front.* For example the people waiting for their turn in railway reservation counter window, payment bill line at the big bazaar cash counter and many other example of real world where the line is maintain for the service. The service for these type of lines are on the bases of first come first serve (FCFS) i.e. the person who comes first for the service is on the *front* of the queue and the person who just arrived for the service or join the queue for the service entered from the *rear* of the queue. Therefore queue provides the service to handle the elements for insertion and deletion on the basis of *First-in-First-out (FIFO)* or *Last-in-Last-out (LILO).* Thus Queue is also called *First-in-First-out (FIFO)* list since the first element in queue will be the first element out of the queue. An important example of a queue in computer science occurs in a time sharing system in which programs with the same priority form a queue while waiting to be executed. The other example which can see more common in computer system is the queue of tasks waiting for the line printer, for access to disk storage. Following figure is a representation of a queue illustrating how an insertion is made to the rightmost element in the queue, and how a deletion consists of deleting the leftmost element in the queue.



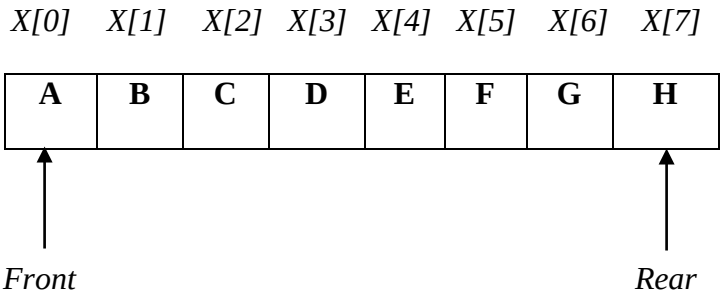
In this case of a queue, the updating operation is restricted to the examination of the last or end element. The size of the queue is fixed and the maximum number of elements can enter in the queue up to the limit of its size. This is called as the ***liner queue.*** A queue of elements of type **A** is a finite sequence of elements of **A** together with the following operations:

1. Initialization a queue to be empty.
2. Determine if a queue is empty or not.

3. Determine if a queue is full or not.
4. Insert a new element after the last element in a queue, if it is not full.
5. Retrieve the first element of a queue, if it is empty.
6. Delete the first element in a queue, if it is not empty.

6.3 REPRESENTATION OF QUEUE

Queues may be represented in the computer in various ways, generally by means of one-way lists or linear arrays. Queues are maintained by a linear array say ***QUEUE*** and two pointer type variables: ***FRONT,*** containing the location of the front element of the queue (the oldest element in the queue or the element which will be first delete); and ***REAR***, containing the location of the rear element of the queue (the newest element in the queue or the recently inserted element in the queue). The condition ***FRONT = REAR = NULL*** will indicate that the queue is empty and ***FRONT = 1, REAR = FRONT = MAX_SIZE*** indicates that the queue is full. These conditions are valid only for the ***linear queue.*** The following figure represents the implementation of queue as an array which is declared to its maximum size as per the requirement of the problem or the number of elements those has to be entered in the queue. The size of the Queue keeps on changing as the elements are either removed from the front end or added at the rear end but the size of the array will remain fixed.



6.4 INSERTION AND DELETION IN THE LINEAR QUEUE

Now we formulate algorithms for the insertion of an element of an element to and the deletion of an element from a queue. We consider a linear array of arbitrary size. This array is assumed to consist of a large number of elements, enough to be sufficient to handle the elements of the linear queue. This array representation of a queue consists with two pointer type variables *Rear (R)* and *Front (F)*. The algorithms of insertion and deletion from the linear queue which is implemented with linear array ***(Q)*** can describe as:

Function *QINSERT (Q, F, R, Y):* [Given *F* and *R*, pointers to the front and rear elements of a queue, a queue ***Q*** consisting of ***N*** elements, and an element ***Y***, this function inserts ***Y*** at the rear of the queue.]

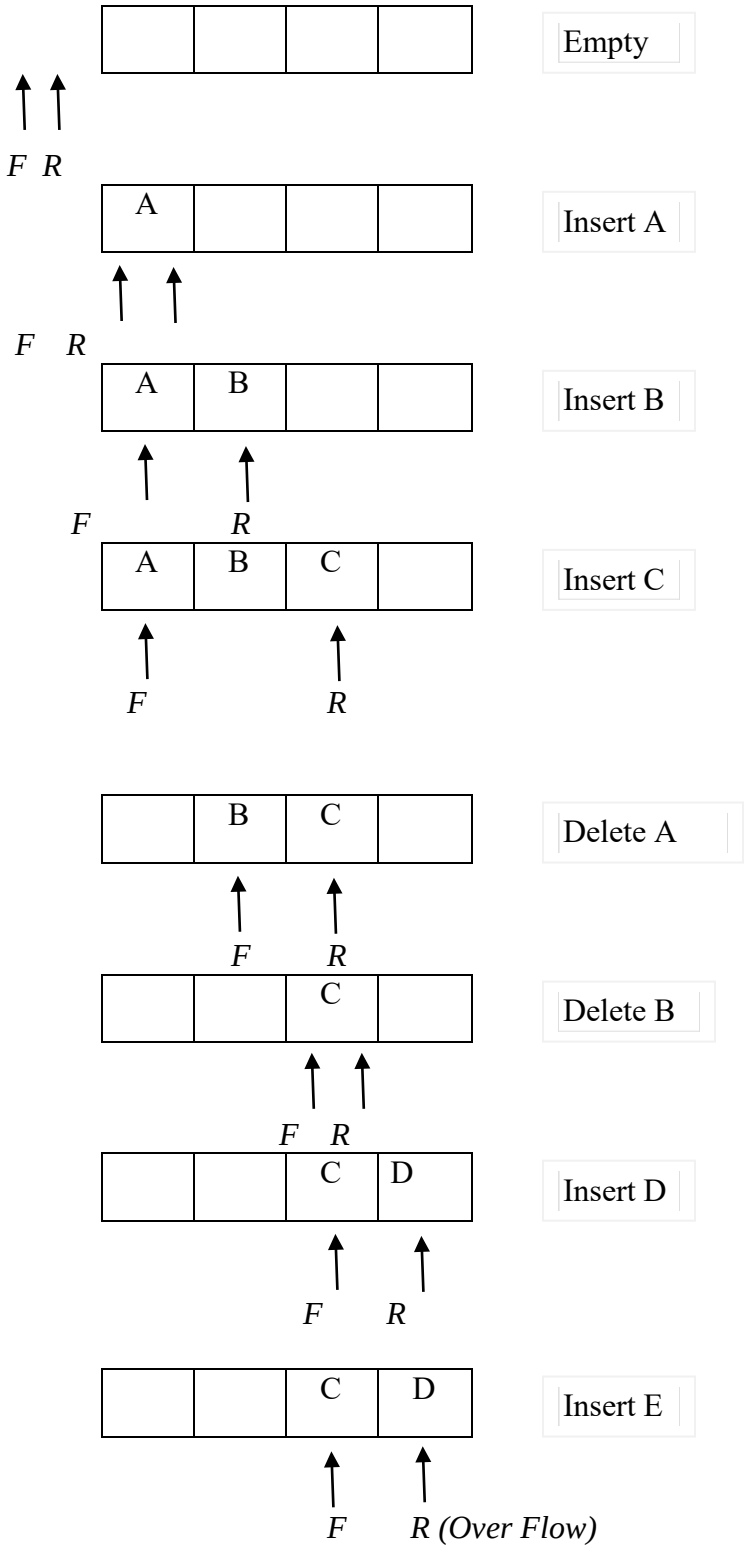
```
If( R>=N)     /* check for the overflow condition */
    Printf ("Overflow"); return;
Else
{
R = R + 1;  /*Increment rear pointer */
Q[R] = Y;   /* insert element */
If (F == 0)  /* set the front pointer */
F = 1;
Return;
}
```

Function *QDELETE (Q, F, R):* [Given *F* and *R*, pointers to the front and rear elements of a queue respectively and the queue ***Q*** to which they correspond, this function deletes and returns the last element of the queue. ***Y*** is a temporary variable.]

```
If (F==0)     /* check for the underflow condition */
    Printf ("underflow"); return 0;  /* 0 denotes the empty queue */
Else
{
Y = Q [F];  /* delete element */
If (F == R)
F = R = 0;
Else
F = F + 1;  /* increment front pointer */
Return Y;
}
```

6.5 EXAMPLE

Consider an example where the size of the queue is four elements. Initially, the queue is empty. It is required to insert symbols 'A', 'B' and 'C', delete 'A' and 'B', and insert 'D' and 'E'. The trace for the insertion and deletion algorithms can represent as follows for the given Queue:



Note : *R will increment only when there is no overflow i.e. if (R>=N) it simply print "Overflow"*

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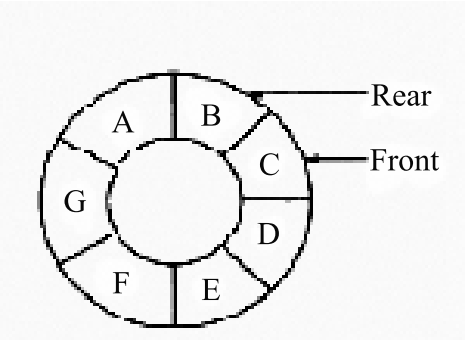
3.6 CIRCULAR QUEUE

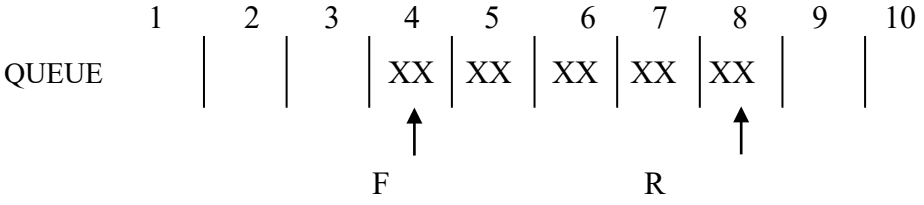
A more suitable method of representing a queue, which prevents an excessive use of memory, is to arrange the elements *Q [1], Q [2],……..Q [n]* in a circular fashion with *Q [1]* following *Q [n]*. Thus circular queues are the queues implemented in circular form rather than in a straight line. Hence circular queues overcome the problem of unutilized space in linear queue implemented as an array. In the array implemented there is a possibility that the queue is reported full even though slots or space in the queue are empty (since *Rear* has reached to the end of array). The concept of circular queues can also understand as follows:

Suppose an array ***Q*** of ***n*** elements is used to implement a circular queue. If we go on adding elements to the queue we may reach ***Q [n-1].*** We cannot add any more elements to the queue since the end of the array has been reached. Instead of reporting the queue is full, if some elements in the queue have been deleted then there might be empty slots at the beginning of the queue. In such case these slots would be filled by new elements added to the queue. Thus, just because we have reached the end of the array, the queue would not be reported as full. The queue would be reported full only when all the slots in the array are occupied. The circular queue can view in the following figure as:

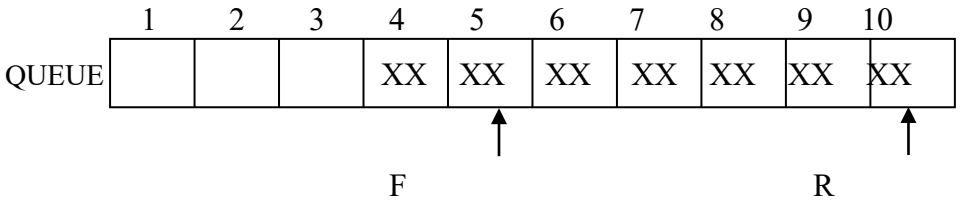


The Circular Queue

Hence in the linear arrangement of the queue always considers the elements in forward direction. In the insertion and deletion algorithms for the linear queue, we had seen that, the pointers *front* (***F)*** and *rear* ***(R)*** are always incremented as and when we delete or insert element respectively. Suppose in a queue of 10 elements front points to 4th element and rear points to 8th element as follows.



When we insert two more elements then the array will become



Later, when we try to insert some elements, then according to the logic when REAR is 10 then it encounters an overflow situation. But there are some elements are left blank at the beginning part of the array. To utilize those left over spaces more efficiently, a circular fashion is implemented in queue representation. The circular fashion of queue reassigns the rear pointer with 1 if it reaches 10 and beginning elements are free and the process is continued for deletion also. Such logic is used for insertion and deletion in Circular Queue.

6.7 INSERTION AND DELETION IN THE CIRCULAR QUEUE

Now we formulate algorithms for the insertion of an element of an element to and the deletion of an element from a circular queue. We consider a array of arbitrary size. This array is assumed to consist of a large number of elements, enough to be sufficient to handle the elements of the linear queue. This array representation of a queue consists with two pointer type variables *Rear (R)* and *Front (F)*. The algorithms of insertion and deletion from the circular queue which is implemented with array ***(Q)*** can describe as:

Function CQINSERT *(Q, F, R, Y):* [Given *F* and *R*, pointers to the front and rear elements of a circular queue, ***F*** and ***R,*** a queue ***Q*** consisting of ***N*** elements, and an element ***Y***, this function inserts ***Y*** at the rear of the queue.]

```
If (R==N)     /* Reset rear pointer */
    R = 1;
Else
R = R + 1;
If (F==R)  /* Check over flow condition */
{
Printf ("Over Flow")
Return
}
Else
{
Q[R] = Y;   /* insert element */
```

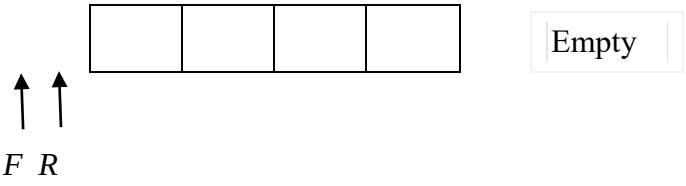
```
If (F == 0)  /* set the front pointer */
F = 1;
Return;
}
```

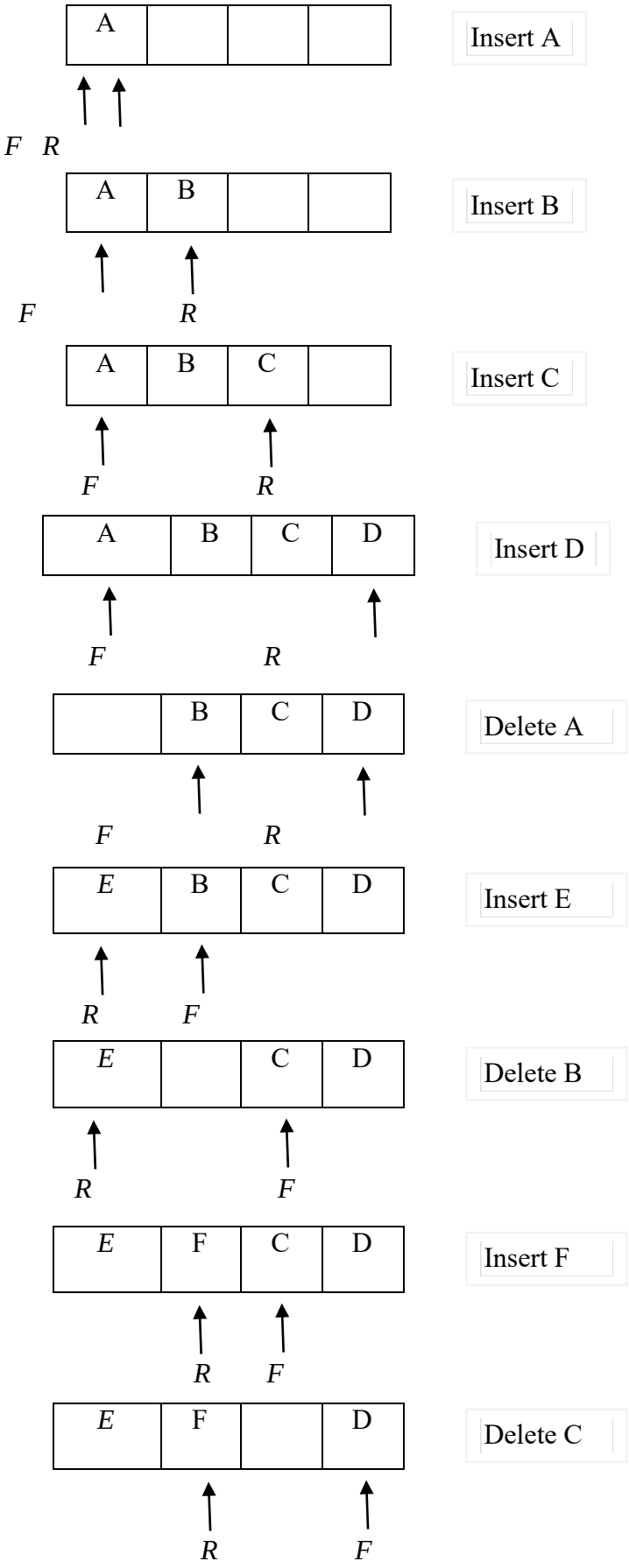
Function *CQDELETE (Q, F, R):* [Given *F* and *R*, pointers to the front and rear of a circular queue, respectively, and a Queue ***Q*** consisting of ***N*** elements, this function deletes and returns the last element of the queue. ***Y*** is a temporary variable.]

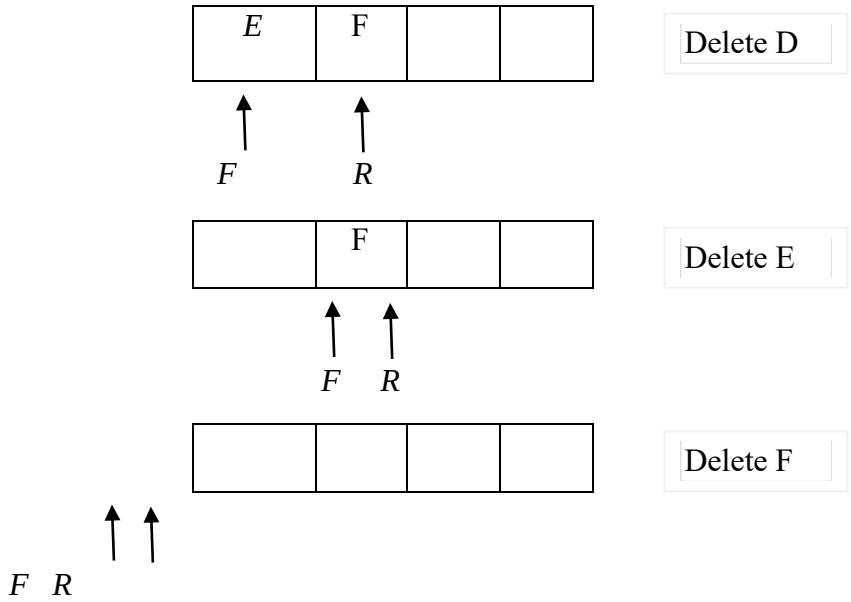
```
If (F==0)     /* check for the underflow condition */
    Printf ("underflow"); return 0;  /* 0 denotes the empty queue */
Else
{
Y = Q [F];  /* delete element */
If (F == R)  /* Check whether the queue is empty */
{
F = R = 0;
Return (Y); }
Elseif (F == N)
F = 1;
else
F = F + 1;  /* increment front pointer */
Return Y;
}
```

6.8 EXAMPLE

Consider an example of a circular queue that contains a maximum of four elements. It is required to perform a number of insertion and deletion operations on an initially empty queue. It is required to insert symbols 'A', 'B', 'C' and 'D', delete 'A', insert 'E', delete 'B', insert 'F', delete 'C', 'D', E' and 'F'. The trace for the insertion and deletion algorithms can represent as follows for the given Queue:




A
Insert A
F R
A B
Insert B
F R
A B C
Insert C
F R
A B C D
Insert D
F R
B C D
Delete A
F R
E B C D
Insert E
R F
E C D
Delete B
R F
E F C D
Insert F
R F
E F D
Delete C
R F



6.9 TYPES OF QUEUES

There are two types of Queues

- Priority Queue
- Double Ended Queue

Priority Queue

A priority queue is a collection of elements such that each element has been assigned a priority value such that the order in which elements are deleted and processed comes from the following rules:

1. An element of higher priority is processed before any element of lower priority.
2. Two elements with the same priority are processed according to the order in which they were added to the queue.

There are various ways of maintaining a priority queue in memory. One is using one way list. The sequential representation is never preferred for priority queue. We use linked Queue for priority Queue.

Double Ended Queue

A Double Ended Queue is in short called as Deque (pronounced as Deck or dequeue). A deque is a linear queue in which insertion and deletion can take place at either ends but not in the middle.

There are two types of Deque.

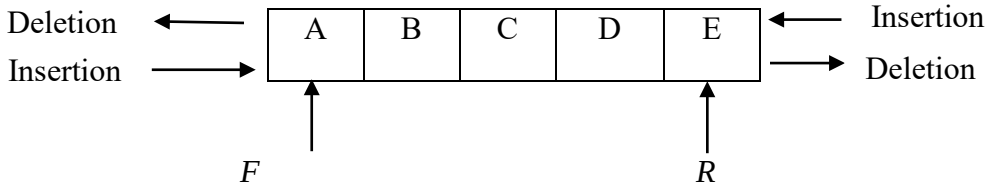
1. Input restricted Deque
2. Output restricted Deque

- A Deque which allows insertion at only at one end of the list but allows deletion at both the ends of the list is called Input restricted Deque.
- A Deque which allows deletion at only at one end of the list but allows insertion at both the ends of the list is called Output restricted Deque.

The two possibilities that must be considered while in serting or deleting elements into the queue are:

- When an attempt is made to insert an element into a deque which is already full, an **over flow** occurs.
- When an attempt is made to delete an element from a deque which is empty, **underflow** occurs.

The Deque can represent as follows:



6.10 SUMMARY

Queue is linear data structure in which the element is inserted from one end of the queue. Queue is also called First-in-First-out (FIFO) list since the first element in queue will be the first element out of the queue. In this unit we have described the insertion and deletion of an element in the linear queue with the condition of Overflow and underflow. The same operations are performed for circular queue and again the conditions of overflow and underflow are explicitly defined. The contents of this unit can be summarized as:

- Queue is a linear data structure that permits insertion of new element at one end and deletion of an element at the other end. Queue is also referred to as first-in-first-out (FIFO) list.
- Two pointers are used in the queue i.e. rear and front. From rear the elements are inserted and from front the elements are deleted but one at a time.
- Circular queues are the queues implemented in circle rather than a straight line.
- Conditions of overflow and underflow are different for linear queue and circular queue.
- Deques are the queues in which elements can be added or removed at either end but not in the middle.

- An input-restricted deque is a deque which allows insertion at only one end but does not allow deletions at both the ends of the list.
- An output-restricted deque is a deque which allows deletions at only one end of the list but allows insertions at both the ends of the list.
- A queue in which it is possible to insert an element or remove an element at any position depending on some priority is called priority queue.
- Queue is a data structure used in many applications like event simulation, job scheduling, etc.

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SELF EVALUATION

Multiple Choice Questions :

1. *"FRONT = REAR"* pointer refer to empty:

   a. Stack

   b. Queue

   c. Array

   d. None of the above

2. A data structure in which insertion and deletion can take place at both ends is called:

   a. Deque b. Stack

   c. Circular Queue d. None of these

3. Using arrays, most efficient implementation of queue is on:

   a. Linear queue b. Priority queue

   c. Circular queue d. None of the above

4. …… form of access is used to add and remove nodes from a queue

   a. LIFO, Last In First Out.

   b. FIFO, First In First Out

   c. LILO, Last in Last Out

   d. Both b and c.

5. New nodes are added to the ……. of the queue.

   a. Front

   b. Back

   c. Middle

   d. Both a and b.

Fill in the Blanks

1. A queue can be defined as a ……. (data type / data structure)

2. The term head of the queue is same as the term……. (front / rear)

3. *FRONT = REAR* pointer refers to……. Queue. (empty / full)

4. A / An…… is a queue in which insertion of an element takes place at both the ends but deletion occurs at one end only. (input-restricted / output restricted)

5. A ----------is a data structure that organizes data similar to a line in the super market, where the first one in the line is the first one out (Linear queue / Circular queue)

State whether True or False

1. A queue can be implemented using a circular array with front and rear indices and one position left vacant.

2. Queue is a useful data structure for any simulation application.

3. A priority queue is implemented using an array of stacks.

4. Queues are often referred to as Last in First out (LIFO) data structure.

5. A deque is a generalization of both a stack and a queue.

Descriptive Questions:

1. Show how a sequence of insertion and removals from a queue represented by a linear array can cause overflow to occur upon an attempt to insert an element into an empty queue.

2. How would you implement a queue of stacks? A stack of queues? A queue of queues? Write routines to implement the appropriate operations of each of these data structures.

3. What is a circular queue? Write a C program to insert an element in the circular queue. Write another C function for printing elements of the queue in reverse order.

4. Given the circular queue of with $F = 6$ and $R = 2$, give the values of R and F after each operation in the sequence: insert, delete, delete, insert and delete.

Uttar Pradesh Rajarshi Tandon
Open University

Postgraduate Diploma in

Computer Application

PGDCA-107

Data Structure

BLOCK

3

Linked List, Tree and Graph

Curriculum Design Committee

Dr.P.P.Dubey — **Coordinator**
Director, School of Agri. Sciences,
UPRTOU, Prayagraj

Prof. U. N. Tiwari — **Member**
Dept. of Computer Science and Engg.,
Indian Inst. Of Information Science and Tech.,
Prayagraj

Prof. R.S. Yadav, — **Member**
Dept. of Computer Science and Engg.,
MNNIT, Allahabad, Prayagraj

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Dept. of Computer Science,
Baranas Hindu University, Varanasi

Mr. Prateek Kesrwani — **Member Secretary**
Academic Consultant-Computer Science
School of Science, UPRTOU, Prayagraj

Course Design Committee

Prof. U. N. Tiwari — **Member**
Dept. of Computer Science and Engg.,
Indian Inst. Of Information Science and Tech.,
Prayagraj

Prof. R.S. Yadav, — **Member**
Dept. of Computer Science and Engg.,
MNNIT, Prayagraj

Prof. P. K. Mishra — **Member**
Dept. of Computer Science,
Baranas Hindu University, Varanasi

Faculty Members, School of Sciences

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Dr. R . P . S ingh, A cademic C onsultant (Bio-Chemistry), School of Science, UPRTOU, Prayagraj

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Dr. S . S . T ripathi, A cademic C onsultant (Maths), School of S cience, UPRTOU, Prayagraj

Course Preparation Committee

Prof. Manu Pratap Singh, **Author**

Dept. of Computer Science

Dr. B. R. Ambedkar University, Agra-282002

Dr. Ashutosh Gupta **Editor**

Director, School of Sciences,

UPRTOU, Prayagraj

Prof. U. N. Tiwari **Member**

Dept. of Computer Science and Engg.,

Indian Inst. Of Information Science and Tech.,

Prayagraj

Prof. R.S. Yadav **Member**

Dept. of Computer Science and Engg.,

MNNIT, Allahabad, Prayagraj

Prof. P. K. Mishra **Member**

Dept. of Computer Science

Baranas Hindu University, Varanasi

Dr. Dinesh K Gupta, **SLM Coordinator**

Academic Consultant- Chemistry School of Science, UPRTOU, Prayagraj

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BLOCK INTRODUCTION

This block will cover the one linear but non-contiguous data structure i.e. linked list and two non-linear & non-contiguous data structure i.e. Tree and Graph. All of these three data structures support dynamic memory allocation. The operations of insertion and deletion are explained with the help of examples for single linked list, circular linked list and doubly linked list. The applications of linked list data structure are discussed specifically for representation of polynomial. Enough number of examples is discussed to show the operations in linked list.

Tree data structure is also discussed in full detail. The basic terminology of tree and its representation is explained with suitable examples. The concepts of binary tree, complete binary tree, binary search tree, AVL tree, B-tree are illustrated with suitable examples. The traversing for the tree is explained with in-order, pre-order and post-order manner. The iterative and recursive algorithms for these traversing are also described. The insertion and deletion of an element from the binary tree and binary search tree is also explained with suitable examples. Threaded tree is explained and its representation is considered with example. The operations of insertion and deletion are defined for B-tree and AVL-tree also.

Graph is explained with its used terminology. Various methods for graph representation are covered like matrix representation and linked list representation. The adjacency matrix, path matrix and reach matrix are explained with suitable examples. The acyclic graph and directed graph are also covered in this block. The two basic search techniques of searching i.e. Breadth first search and depth first search for the graph is also discussed and explained with examples. The concept of spanning tree and minimum spanking tree is stated with the Kruskal's and prim's algorithm for minimum spanning tree construction form the given weighted di graph. The Shortest path algorithms like bellman Ford, Dijkstra's and Floyd-Warshall are explained with examples. The topological sort for the graph is also covered with example.

This block will help you to realize the concept of non-primitive data structures and illustrate you about the application of these important data structure in the computer organization and for processing of various common operations of system software.

UNIT-7 LINKED LIST

Structure

7.0 INTRODUCTION

A list can be defined as a collection of elements. We can add, search, or delete elements in a list. The list in maintained either with array or linked list. This unit introduces the concept of another important linear but non-contiguous data structure i.e. linked list which is a linear collection of data elements called nodes, which pointing to the next node by means of pointers. This unit starts with the representation and implementation of the single liked list with operation of insertion and deletion of the element in the liked list on various locations i.e. at first, last and at middle. It gives the implementation of stack and queue with the help of single liked list. The concept and implementation of circular liked list & doubly liked list is presented with the operation of insertion and deletion of an element on different locations. The application of linked list

is presented for representation of a polynomial and shows the operation of addition in between two polynomials by using linked list. In the last the concept of garbage collection is introduced. Linked lists overcome the drawbacks of arrays. In a linked list the number of elements need not be predetermined; more memory can be allocated and released during processing. It supports the dynamic memory allocation mechanism to make insertion and deletion easier.

7.1 OBJECTIVES

After going through this unit, you should be able to:

- Understand the concept of linked list and its representation.
- Implementation of single linked list and performing for the operation of insertion and deletion for an element in the existing linked list on various locations.
- Implementation of stack and queue with single linked list.
- Understand the concept of circular linked list and its representation.
- Implementation of circular linked list and performing for the operation of insertion and deletion for an element in the existing circular linked list on various locations.
- Implementation of doubly linked list and performing for the operation of insertion and deletion for an element in the existing doubly linked list on various locations.
- Understand the application of linked list for the representation of polynomial and performing the operation of addition for two polynomials using linked list.
- Understand the concept of garbage collection.

7.2 DEFINITION OF LINKED LIST

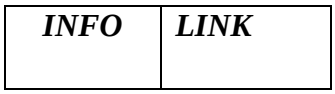
A structure involved in many data processing activities is the ordered list of data items, like alphabetical lists of names. This type of data processing is conveniently performed with Array. In Array there is a linear relationship; between the data elements those are stored in memory at contiguous location with static memory requirement. The address of any element in the array can be easily computed but it is very difficult to insert and delete any element in an array. Usually, a large block of memory is occupied by an array which may not be in use and it is difficult to increase the size of an array, if required. There is another way also for storing the ordered list is to have each element in a list contain a field called a **link** or **pointer**, which contains the address of the next element in the list. Thus it provides the non contiguous memory allocation but in liner relationship.

Hence the successive elements in the list need not occupy adjacent space in memory. This type of data structure is called a **linked list**. Thus, Linked list is the most commonly used data structure used to store similar type of data in memory. The elements of a linked list are not stored in adjacent memory locations as in arrays.

Therefore, a **linked list** or one-way list is a liner collection of data elements, called **nodes**, where the linear order is given by means of pointers. That is, each node is divided into two parts:

1. **First part**: It contains the information of the element ***(Info)***
2. **Second part** : It contains the address of the next node in the list ***(Link)***

Any **node** of the linked list can represent as:



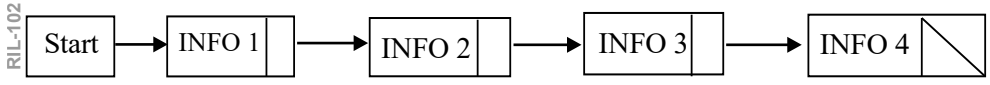
Node

In this type of list representation with linked list which is containing **nodes** for the presentation of elements a pointer is used to represent the address of the next element of the linked list. A pointer to the starting of the linked list i.e. for the first node or **head** of the linked list is used to gain access to the list itself and the end of the list is denoted by a **NULL** pointer. In 'C' language the structure is used to implement a single linked list as:

```
Struct node
{
        Int INFO;
        Struct node *LINK;    }
```

The structure declared for linear linked list holds two members, an integer type variable **INFO** which holds the elements and another member of type **node** which has the variable **LINK,** which stores the address of the next node in the list.

Now we consider an example for the representation of a linked list of 4 nodes as shown in the following figure. Each node is considering two parts. The left part represent the information part of the node, which may contain an entire record of data items. The right part represents the Link part i.e. a pointer field which contains the address of the next node. The pointer of the last node contains the *NULL* pointer, which is any invalid address.



(*NULL Pointer)*

7.3 DYNAMIC MEMORY ALLOCATION FOR LINKED LIST

The 'C' language requires the number of elements in an array at compile time. This may cause wastage of memory space. Such situation can take care by using the concept of dynamic memory allocation concepts. Dynamic memory techniques allow us to allocate additional memory space or to release unwanted space at the time execution. This may cause minimum wastage to memory space with respect to the static memory allocation at compile time. The C language supports the following functions for allocating and free memory during the execution of the program:

- Malloc () : It allocates requested size of bytes
- Calloc () : It allocates space for an array of elements
- Free () : It Frees previously allocated space
- Realloc () : It modifies the size of previously allocated space.

7.4 CREATING THE LINKED LIST IN 'C'

The linked list can create in 'C' language by using pointers and dynamic memory allocation functions such as **malloc().** The **head** pointer is used to create and access unnamed nodes. The following code segment is used in 'C' for creating the linked list:

```
Struct linked_list
{
Int INFO;
Struct linked_list *next;   }
Typedef struct linked_list node;
Node *head;
Head = (node*) malloc (size of (node));
```

Thus the above segment of code obtains memory to store a node and assigns its address to **head** which is a pointer variable.

7.5 OPERATION ON LINKED LIST

We can perform the following operations on the linked list:

1. Traversing a Linked List

Let we have the linked list **LIST** in memory with its two fields ***INFO* and *LINK*** with **START** pointing to the first element and ***NULL*** indicating the end of ***LIST.*** Suppose we want to traverse **LIST** and print the element of the list in order to process each node exactly once. Our traversing algorithm uses a pointer varivable ***PTR*** which points to the node that is currently being processed. Accordingly, **LINK[PTR]** points to the next node to be processed. The assignment ***PTR = LINK[PTR]*** moves the pointer to the next node in the list. The algorithmic steps for the traversing process are as follows:

Algorithm (Traversing a Linked List) :

[Initialize ***PTR*** and then process ***INFO [PTR],*** the information at the first node. Update ***PTR*** by the assignment ***PTR = LINK[PTR],*** and then process ***INFO[PTR]***, the information at the second node and so on until ***PTR = NULL,*** which signals the end of the list.]

1. ***PTR = START*** *[Initialize pointer* ***PTR]***
2. *Repeat steps 3 and 4 while* ***PTR ≠NULL***
3. *Print* ***(INFO [PTR])***
4. ***PTR = LINK [PTR]*** *[Update pointer]*

   [End of step 2 loop]
5. *Return*

2. Count the nodes in a Linked List

Let we have the linked list **LIST** in memory with its two fields ***INFO* and *LINK*** with **START** pointing to the first element and ***NULL*** indicating the end of ***LIST.*** Suppose we want to count the number of nodes in the linked list by traversing the **LIST** in order to process each node exactly once. Our counting algorithm uses a pointer varivable ***PTR*** which points to the node that is currently being processed. Accordingly, **LINK[PTR]** points to the next node to be processed. The assignment ***PTR = LINK[PTR]*** moves the pointer to the next node in the list. The algorithmic steps for the traversing process are as follows:

Algorithm COUNT (*INFO, LINK, START, NUM)*

[Initialize ***PTR*** and ***NUM*** and then process ***INFO [PTR],*** the information at the first node. Update ***PTR*** by the assignment ***PTR = LINK[PTR],*** and then process ***INFO[PTR]***, and increment the ***NUM*** by one each time of this processing until ***PTR = NULL,*** which signals the end of the list.]

1. ***PTR = START*** *and* ***NUM = 0;*** *[Initialize pointer* ***PTR*** *and* ***NUM]***

2. *Repeat steps 3 and 4 while* ***PTR ≠NULL***

3. ***NUM = NUM + 1;*** *[Increase* ***NUM*** *by 1]*

4. ***PTR = LINK [PTR]*** *[Update pointer]*

5. *Print ("The number of nodes in the linked list are = ",* ***NUM)***

   [End of step 2 loop]

6. *Return*

3. Searching a Linked List

Let we have the linked list **LIST** in memory with its two fields ***INFO*** **and** ***LINK*** with **START** pointing to the first element and ***NULL*** indicating the end of ***LIST.*** We have given an ***ITEM*** of information. The process here is for finding the location ***LOC*** for the node where ***ITEM*** first appears in ***LIST***.

Algorithm (Searching a Linked List):

[Initialize ***PTR*** and then process ***INFO [PTR],*** the information at the first node. Update ***PTR*** by the assignment ***PTR = LINK[PTR],*** and then process ***INFO[PTR]***. Now compare the content ***INFO[PTR]*** of each node with ***LOC***, one by one by updating the pointer ***PTR*** by ***PTR = LINK[PTR]*** until ***PTR = NULL,*** which signals the end of the list. Here we have the two terminating condition i.e. the end of the linked list or the item ***LOC*** is found i.e. ***INFO[PTR]=ITEM***.

Search(*INFO, LINK, START, ITEM, LOC)*

1. ***PTR = START*** *[Initialize pointer* ***PTR*****]**

2. Repeat steps 3 while ***PTR ≠NULL***

3. *If (****ITEM == INFO[PTR]) {***

   LOC = PTR; *Return* ***(PTR); }***

   Else

   PTR = LINK [PTR] *[****PTR*** *now points to the next node]*

   [End of Step 2 loop]

4. ***LOC = NULL*** *[Search is unsuccessful.]*

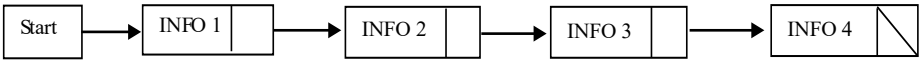
5. *Exit.*

7.6 INSERTION INTO A LINKED LIST

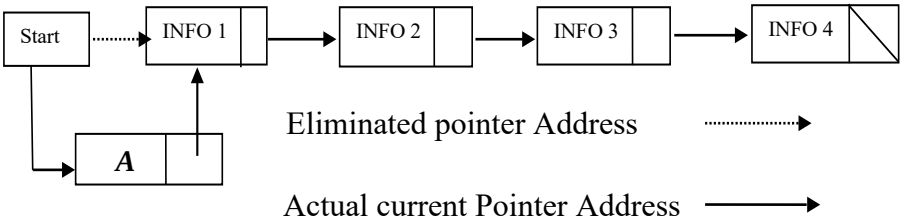
Let ***LIST*** be a linked list on any arbitrary nodes. An element ***A*** is to be inserted into the list. There may be the following three conditions in which the given element can insert in the existing list:

1. The new element ***A*** can insert at the beginning of the list as the first node.
2. The new element ***A*** can insert at the end of the list.
3. The new element ***A*** can insert in between any two already existing nodes in the list.

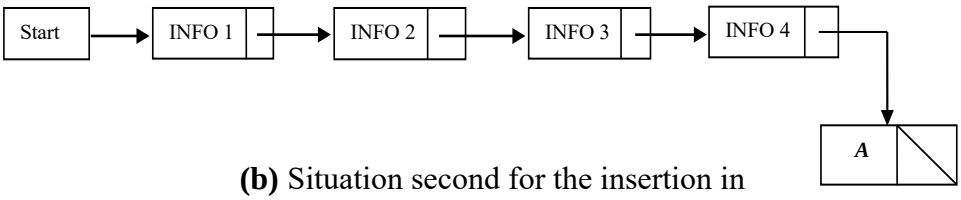
All the three locations for the insertion of given new element ***A*** into the existing linked list can view diagrammatically as:



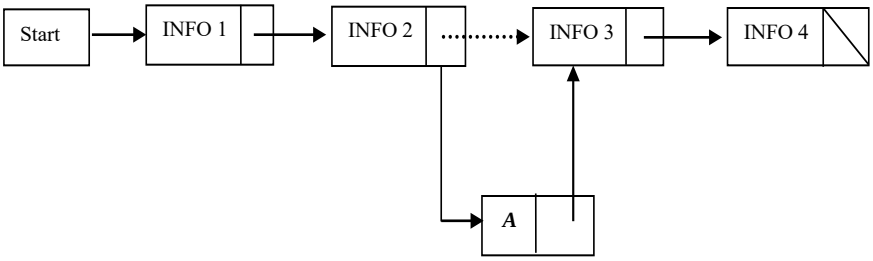
Existing Linked List



(a) Situation First for the insertion in existing List



(b) Situation second for the insertion in existing List



Eliminated pointer Address ⋯⋯▶

Actual current Pointer Address ⟶

(c) Situation Third for the insertion in existing List

Now we present the algorithm of insertion on these three locations in the given linked list. Thus we discuss three insertion algorithms for the following cases:

- The first one inserts a node at the beginning of the list
- The second one inserts a node into after the node with a given location.
- The third one inserts a node into the last or into a sorted list.

In all the following algorithms we consider a linked list i.e. ***LIST (INFO, LINK, START, AVAIL)*** and that the variable ***ITEM*** contains the new information to be added to the list. Since our insertion algorithms will use a node in the ***AVAIL*** list, all of the algorithms will include the following steps:

(a) Check the condition for ***overflow***. This condition will check to examine the ***AVAIL*** list i.e. if ***AVAIL = NULL*** then it shows the condition of ***overflow***.

(b) Removing the first node from the ***AVAIL*** list. Using the variable ***NEW*** to keep track of the location of the new node, this step can be implemented as:

NEW = AVAIL, AVAIL = LINK [AVAIL]

(c) Copying new information into the new node as:

INFO [NEW] = ITEM

The algorithms for these three cases are described as:

(i) **Insertion at the beginning of a List**

This algorithm inserts a new element at the beginning of the given linked list:

INSFIRST (INFO, LINK, START, AVAIL, ITEM)

[This algorithm will insert the given ***ITEM*** as the first node in the list]

If ***(AVAIL == NULL)*** */* Check the condition for overflow */*

{

*Printf (‘**OVERFLOW**’);*

```
        Return;
    }
Else {
        NEW = AVAIL;
        AVAIL = LINK [AVAIL];      /* remove first node from AVAIL list */
        INFO[NEW] = ITEM;          /* Copy new data into new node */
        LINK [NEW] = START;        /* new node to point the original first node */
        START = NEW;               /* Change START so it points to the new node */
        }
Return (NEW);
```

(ii) **Insertion at end of a List**

This algorithm inserts a new element at the end or at the last of the given linked list:

INSFIRST (INFO, LINK, START, AVAIL, ITEM)

[This algorithm will insert the given ***ITEM*** as the last node in the list]

```
If (AVAIL == NULL)        /* Check the condition for overflow */
    {
        Printf ('OVERFLOW');
        Return;
    }
Else {
        NEW = AVAIL;               /* obtain address of next free node */
        AVAIL = LINK [AVAIL];      /* re remove first node from AVAIL list */
        INFO [NEW] = ITEM;         /* Copy new data into new node */
        LINK [NEW] = NULL          /* new node to point the last node of the list */
```

```
        If (START == NULL)          /* check for the list that is list
empty? */
        Return (NEW);
        TEMP = START                /* Initiate search for the last
node */
         while (LINK [TEMP] != NULL)
        {
        TEMP = LINK [TEMP]
        }
        LINK [TEMP] = NEW           /* Set LINK field of last node
to NEW */
        Return (START)
      }
```

(i) **Insertion after a given Node of a List**

This algorithm inserts a new element after the given node of the given linked list:

INSORDER (INFO, LINK, START, AVAIL, ITEM)

[This algorithm inserts the given node ***ITEM*** into ***LIST*** on the given location i.e. ***LOC*** is the location of a node say ***A***. Thus ***ITEM*** follows node ***A***. Let ***N*** denote the new node (whose location is ***NEW***). If ***LOC=NULL***, then ***N*** is inserted as the first node in ***LIST***. Let the node ***N*** point to node ***B*** (which originally followed node ***A***) by the statement, ***LINK [NEW] = LINK [LOC]*** and we let node ***A*** point to the new node ***N*** by the statement, ***LINK [LOC] = NEW;***

```
    If (AVAIL == NULL)      /* Check the condition for overflow
    */
    {
        Printf ('OVERFLOW');
        Return;
      }
Else {
        NEW = AVAIL;            /* obtain address of next free
node */
        AVAIL = LINK [AVAIL];   /* re remove first node from
    AVAIL list */
```

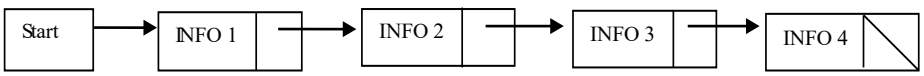
```
        INFO [NEW] = ITEM;          /* Copy new data into new
    node */
        If (LOC == NULL)            /* check for the list that is list
empty? */
        {
        LINK [NEW] = START;
        START = NEW;                /* Insert as first node */
         }
        Else
        {
         LINK [NEW] = LINK [LOC];
         LINK [LOC] = NEW;          /* Insert the node after LOC
        }
      }
        Return (START);
```

7.7 DELETION FORM THE LINKED LIST

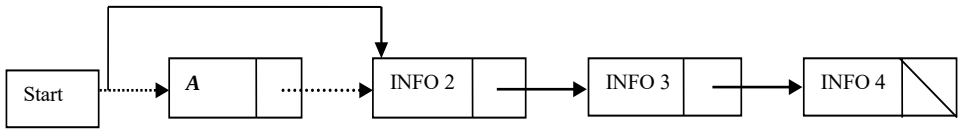
Let ***LIST*** be a linked list on any arbitrary nodes. An element ***A*** is an existing node of the linked list which has to be deleted from the list. There may be the following three conditions in which the given element can insert in the existing list:

1. The node of the element ***A*** is the first node of the linked list which has to be deleted.
2. The node of the element ***A*** is the last node of the linked list which has to be deleted.
3. The node of the element ***A*** is the any middle node of the linked list which has to be deleted. A specific location is given for a node and that node which is at that location should delete.

All three situations for the deletion of given node of element ***A*** into the existing linked list can view diagrammatically as:



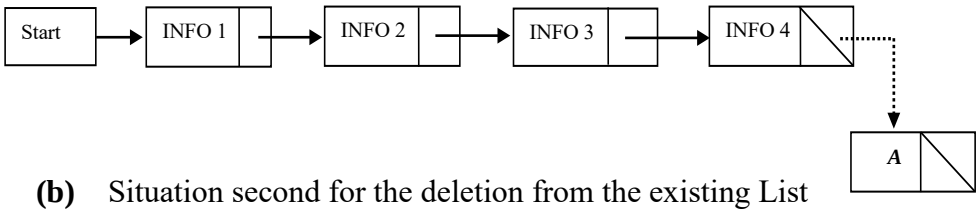
Existing Linked List



Eliminated pointer Address

Actual current Pointer Address

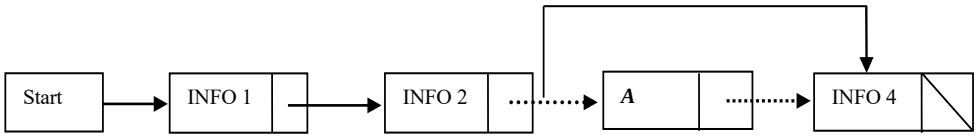
(a) Situation First for the deletion from the existing List



(b) Situation second for the deletion from the existing List

Eliminated pointer Address

Actual current Pointer Address



Eliminated pointer Address

Actual current Pointer Address

(c) Situation Third for the deletion from the existing list

Now we present a general algorithm for deleting an element from the given linked list. This algorithm encompasses all the three situations as described in above mentioned figures for deletion.

Algorithm DELTE (X, FIRST): [Given ***X*** and ***FIRST*** pointer variables whose values denote the address of a node in a linked list and the address of the first node in the linked list, respectively, this procedure deletes the node whose address is given by ***X***. ***TEMP*** is used to find the desired node, and ***PRED*** keeps track of the predecessor of ***TEMP***. ***FIRST*** is changed only when X is the first element of the list.]

```
If (FIRST == NULL)      /* Check the condition for empty list
*/
{
    Printf ('UNDER FLOW');
```

```
            Return;
          }
    Else {                              /* Initialize search for X */
            TEMP = FIRST;
    While ((TEMP != X) && (LINK (TEMP) != NULL))      /* find
X */
    {
            PRED = TEMP                 /*  Update  predecessor
marker */
            TEMP = LINK (TEMP)          /* Move to next node */
    }
            If ( TEMP != X)             /* end of the list */
            {
            Printf ("Node not found");
            Return;
            }
            If (X == FIRST)         /* X is the first node i.e. the deletion
of the first node */
            FIRST = LINK (FIRST);
            Else
            LINK (PRED) = LINK (X);
    LINK (X) = AVAIL;                   /* Return node to available
area */
    AVAIL = X;
    Return;
    }
```

7.8 COPY OF THE LINKED LIST

Let ***LIST*** be a linked list on any arbitrary nodes. We formulate a algorithm which copies a linked list into the another linked list. A general algorithm to copy a linked list is as follows:

Algorithm COPY (FIRST): [Given ***FIRST,*** a pointer to the first node in a linked list, this algorithm makes a copy of this list. A typical node in the given list consists of ***INFO*** and ***LINK*** fields. The new list is to contain nodes whose information and pointer fields are denoted by ***FIELD*** and

PTR, respectively. The address of the first node in the newly created list is to be placed in ***BEGIN. NEW, SAVE*** and ***PRED*** are pointer variables.]

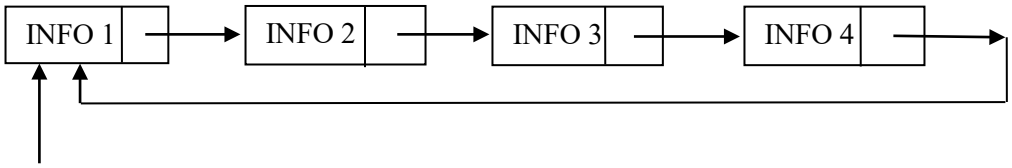
```
If (FIRST == NULL)      /* Check the condition for empty list */
{
    Printf ('EMPTY LIST');
    Return;   }
Else
{
    If (AVAIL == NULL)
    {
    Printf ("Availability stack underflow");
    Return (0);  }
    Else {
    NEW = AVAIL;                /* copy the first node */
    AVAIL = LINK (AVAIL);
    FIELD (NEW) = INFO (FIRST);
    BEGIN = NEW;        }
SAVE = FIRST;                   /* Initialize traversal */
While (LINK (SAVE) != NULL)     /* Move to next node if not at end of list */
{
PRED = NEW;
SAVE = LINK (SAVE);             /* update predecessor and save pointer */
NEW = AVAIL;
AVAIL = LINK (AVAIL);
FIELD (NEW) = INFO (SAVE);      /* copy node */
PTR (PRED) = NEW;
}
PTR (NEW) = NULL;
```

Return (BEGIN); */* Set link of last node and return */*

}

7.9 CIRCULAR LINKED LIST

In the previous discussion we have concerned about linked lists in which the last node of such lists contained the ***null*** pointer. Now we consider a slight different modification of this representation which results in a further improvement in processing. This is accomplished by replacing the null pointer in the last node of a list with the address of its first node. Such a list is called a ***circularly linked linear list*** or simply a ***circular list.*** The following figure illuSTARTes the structure of a circular list:



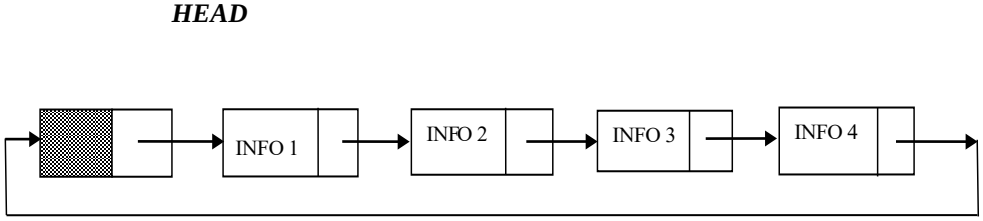
First **A circularly linked list**

Circular lists have certain advantages over singly linked list. These advantages can highlight as follows:

* Accessibility of a node: In a circular list every node is accessible from a given node. That is, from this given node, all nodes can be reached by merely chaining through the list.

* Deletion of a node: In the single linked list, to delete an element in addition to the address X of the node to be deleted it is also necessary to give the address of the first node of the list. This necessity results from the fact that in order to delete X, the predecessor of this node has to be found. To find the predecessor requires that a search be carried out by chaining through the nodes from the first node of the list. This search requirement does not exist for a circular list, since the search for the predecessor of node X can be initiated from X itself.

Beside these advantages the circular link list also has a disadvantage. In circular linked list with if the processing is carried out without some care it is possible to get into an ***infinite loop***. In processing a circular list, it is important that we are able to detect the end of the list. Hence to detect the end of the list we place a special node which can be easily identified in the circular list. This special node is called as the ***list head*** or ***header*** of the circular list. The one of the important advantage of using the **header node** in the circular list is that the list will never be empty because at least one node will always present in the circular linked list. The representation of a circular list with a **header node** can represent graphically as:



A circularly linked list with a head node

Here the variable ***HEAD*** denotes the address of the list head. The ***INFO*** filed in the list head node is not used, which is illuSTARTed by shading the field. An empty list is represented by having:

LINK (HEAD) = HEAD. The following are the various operations performed on a circular header list:

- Traversing a circular header list
- Searching in a circular header list
- Deleting from a circular header list

Now we consider the algorithms for these three operations.

Algorithm (Traversing a circular header Linked List)

[Initialize ***PTR*** and then process ***INFO [PTR],*** the information at the first node. Update ***PTR*** by the assignment ***PTR = LINK [PTR],*** and then process ***INFO [PTR]***, the information at the second node and so on until ***PTR ≠ START.***]

1. ***PTR = LINK [START]*** *[Initialize pointer* ***PTR]***
2. *Repeat steps 3 and 4 while* ***PTR ≠START***
3. *Print* ***(INFO [PTR])***
4. ***PTR = LINK [PTR]*** *[Update pointer]*

   [End of step 2 loop]
5. *Return*

Algorithm (Searching the circular header Linked List)

[This algorithm finds the location ***LOC*** of the node where ***ITEM*** first appears in ***LIST*** or sets ***LOC = NULL.***]

1. ***PTR = LINK [START];*** *[Initialize pointer* ***PTR]***
2. *Repeat while* ***INFO [PTR] ≠ ITEM*** *and* ***PTR ≠START***

   PTR = LINK [PTR]; *[PTR now points to the next node]*

   [End of loop]

3. *If (**INFO [PTR]** == **ITEM**) then*

   LOC = PTR;

   Else

   LOC = NULL;

   [End of if structure]

4. *Exit*

Algorithm CRDELTE (X, FIRST): [Given ***X*** and ***FIRST*** pointer variables whose values denote the address of a node in a linked list and the address of the first node in the linked list, respectively, this procedure deletes the node whose address is given by ***X***. ***TEMP*** is used to find the desired node, and ***PRED*** keeps track of the predecessor of ***TEMP***. ***FIRST*** is changed only when X is the first element of the list.]

```
If (FIRST == NULL)/* Check the condition for empty list */
{
        Printf ('UNDER FLOW');
Return;
    }
  Else {                          /* Initialize search for X */
        TEMP = FIRST;
  While ((TEMP != X) && (LINK (TEMP) != FIRST))
  /* find X */
  {
        PRED = TEMP               /* Update predecessor marker */
        TEMP = LINK (TEMP)        /* Move to next node */
  }
        If ( TEMP != X)           /* end of the list */
        {
        Printf ("Node not found");
        Return;
        }
```

```
    If (X == FIRST)        /* X is the first node i.e. the deletion
of the first node */
        FIRST = LINK (FIRST);
        Else
        LINK (PRED) = LINK (X);
    LINK (X) = AVAIL;                    /* Return node to
available area */
    AVAIL = X;
    Return;
    }
```

7.10 DOUBLY LINKED LISTS

Now we discuss a two – way list, which can be traversed in following two directions:

1. In the usual forward direction from the beginning of the list to the end.
2. In the backward direction from the end of the list to the beginning.

This property of a linked linear list implies that each node must contain two link fields instead of the single link field. The links are used to denote the *Predecessor* and *successor* of a node. The link denoting the *predecessor* of a node is called the *left* link, and that denoting its *successor* its *right* link. A list containing this type of node is called a *doubly linked list* or *two-way list*.

Therefore a two – way list is a linear collection of data structure, called *nodes*, where each *node N* is divided into three parts:

(i) An information field ***INFO*** which contains the data of the element.

(ii) A pointer field ***L*** which contains the location of the preceding node in the list.

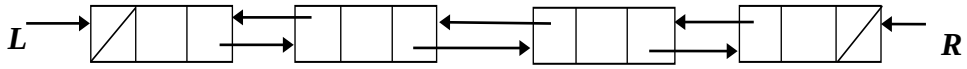
(iii) A pointer field ***R*** which contains the location of the next node in the list.

Any **node** of the doubly linked list can represent as:

LINK (L)	*INFO*	*LINK (R)*

Node

In this linked list there are two *NULL* pointers. One is along the forward pass i.e. with ***R*** link and another is along the backward pass i.e. with ***L*** link. Hence along the forward pass the last node of the list will contain the *NULL* pointer and along the backward pass the first node will contain the *NULL* pointer. Pictorially, such a linear list can be represented diagrammatically as follows:



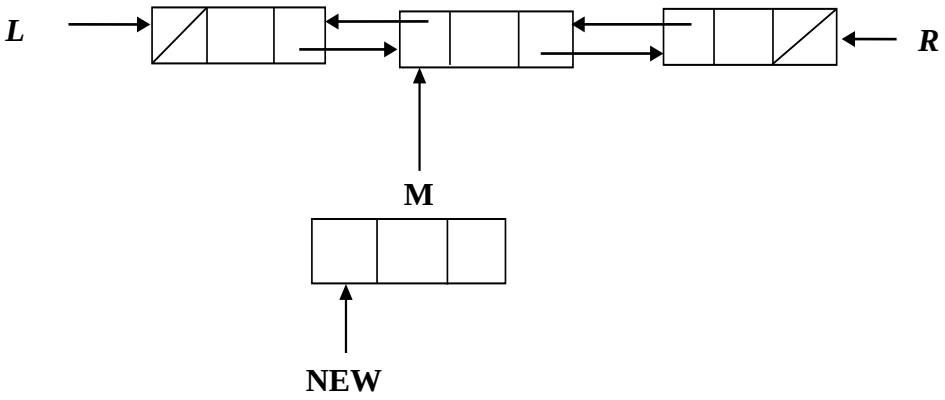
A doubly linked linear list

7.11 INSERTION IN DOUBLY LINKED LIST

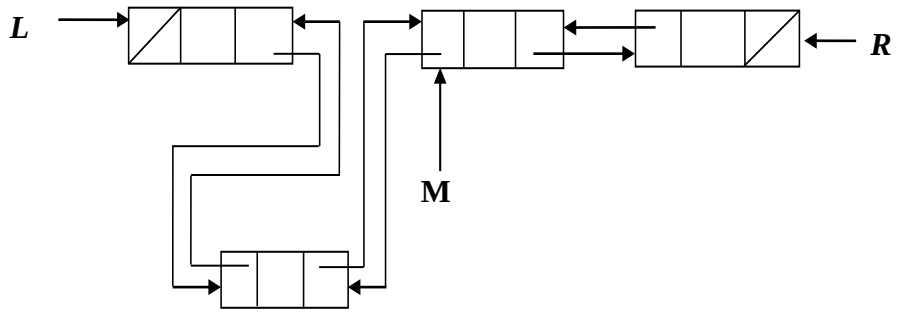
Now we consider the problem of inserting a node into a doubly linked linear list to the ***left*** of a specified node whose address is given by variable ***M***. There are number of cases possible for insertion of an element into the existing doubly linked list. These cases are as follows:

- Insertion in the list which is originally empty. This is denoted by setting both ***L*** and ***R*** pointers to the address of the new node and by assigning a ***NULL*** value to the *left* and *right* link of the node being entered.
- Insertion in the middle of the list.
- The insertion can be made to the left of the left-most in the list, thereby requiring the pointer ***L*** to be changed.

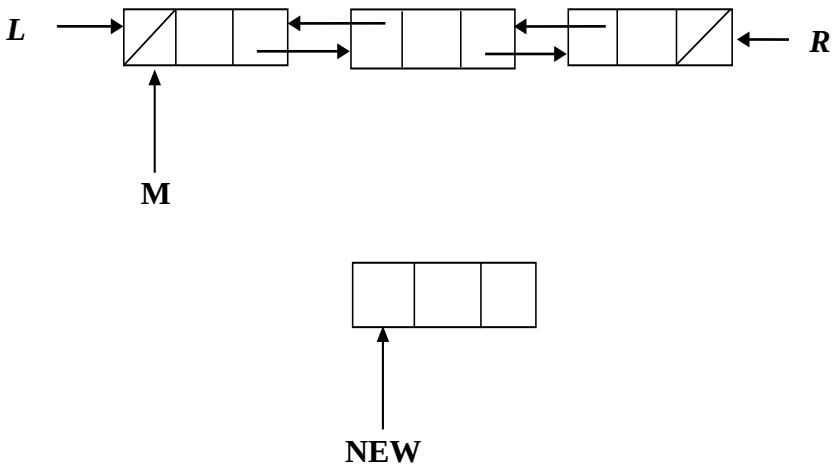
The last two situation of inserting a new element can pictorially represent as:



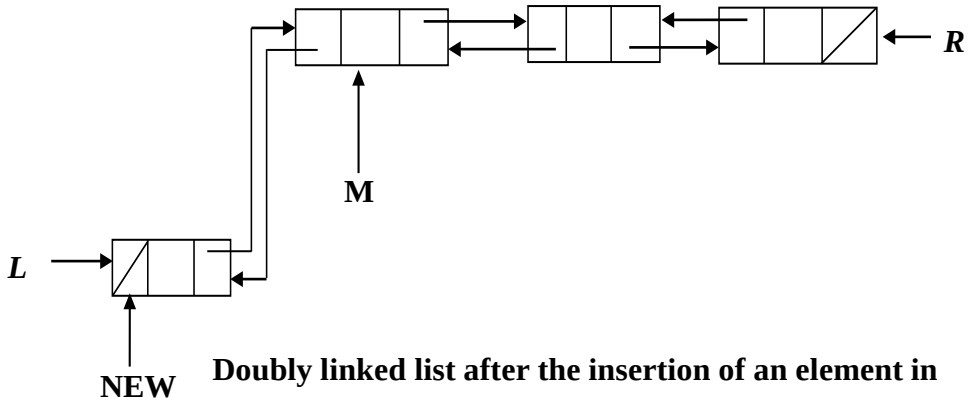
Doubly linked list before insertion of the element



Doubly linked list after the insertion of an element in the middle



Doubly linked list before insertion of the element



Doubly linked list after the insertion of an element in the left most side

Now we describe the general algorithm for inserting a node to the left of a given node in a doubly linked list. The algorithm is stated as:

Algorithm *DOUBINS (L, R, M, X):*

[Given a doubly linked list whose left most and right most node addresses are given by the pointer variables ***L*** and ***R***, respectively, it is required to insert a node whose address is given by the pointer variable ***NEW*** . The left and right links of a node are denoted by ***LPTR*** and ***RPTR***, respectively. The information field of a node is denoted by the variable ***INFO.*** The name of an element of the list is ***NODE***. The insertion is to be performed to the left of a specified node with its address given by the pointer variable ***M***. The information to be entered in the node is contained in ***X***.]

1. *[obtain new node from availability stack]*

   NEW ⇐NODE;

2. *[copy information field]*

   INFO (NEW) = X;

3. *[Insertion into an empty list?]*

   If (R ==NULL)

   {

   LPTR (NEW) = RPTR (NEW) = NULL;

   L = R = NEW;

   Return (NEW);

   }

4. *[Left-most insertion]*

   If (M == L)

   {

   LPTR (NEW) = NULL;

   RPTR (NEW) = M;

   LPTR (M) = NEW;

   L = NEW;

   Return (NEW);

   }

5. *[Insertion in middle]*

   LPTR (NEW) = LPTR (M);

RPTR (NEW) = M;

LPTR (M) = NEW;

RPTR (LPTR (NEW))= NEW;

Return (NEW);

7.12 DELETION IN DOUBLY LINKED LIST

Now we consider the problem of deleting a node from a doubly linked linear list. In this process there is no need of any search for determining the predecessor node of the node to be deleted. In the doubly linked list on giving the address of the node which is to be deleted, the predecessor and successor nodes are immediately known. Therefore doubly linked lists are much more efficient with respect to deletions than singly linked lists.

There are number of possibilities arises for the deletion operation in doubly linked list. These possibilities are as follows:

- If the list contains a single node, then a deletion results in an empty list with the left – most and right-most pointers being set to *NULL*.
- The node being deleted could be the left-most node of the list. In this case the pointer variable ***L*** must be changed.
- The node being deleted could be the Right-most node of the list. In this case the pointer variable ***R*** must be changed.
- The deletion can occur from the middle of the list.

A general algorithm for deleting a node from the doubly linked list is as follows:

Algorithm *DOUBDEL (L, R, OLD):*

[Given a doubly linked list with the addresses of the left most and right most nodes given by the pointer variables ***L*** and ***R***, respectively, it is required to delete the node whose address is given by the pointer variable ***OLD***. The left and right links of a node are denoted by ***LPTR*** and ***RPTR***, respectively.]

1. *[check the condition for underflow]*

   If (R == NULL)

   {

   Printf ("Underflow");

   Return

   }

2. *[Delete the node]*

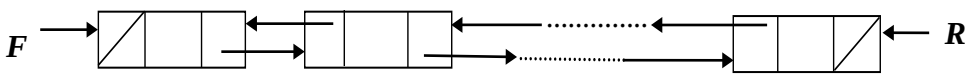
```
If (L == R)                    /* single node in the list */
L = R = NULL;
Else if (OLD == L)             /* Left-most node being deleted */
{
L = RPTR (L);
LPTR (L) = NULL;
}
Else if (OLD == R)             /* Right-most node being deleted */
{
R = LPTR (R);
RPTR (R) = NULL;
}
Else
{
RPTR (LPTR (OLD)) = RPTR (OLD);
LPTR (RPTR (OLD)) = LPTR (OLD);
}
```

3. *[Return deleted node]*

```
Return (OLD);
```

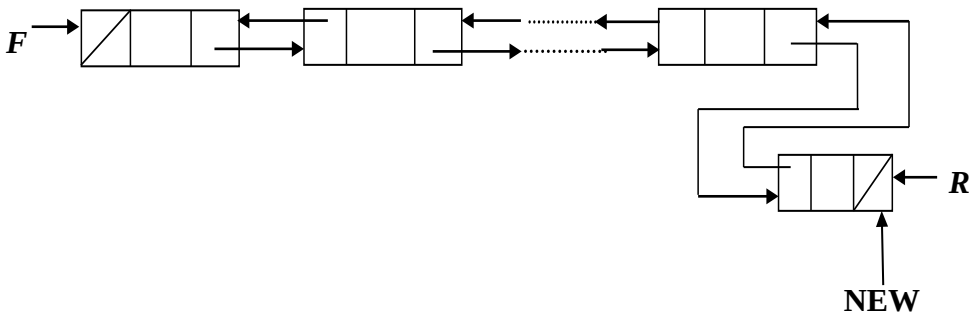
7.13 DOUBLY LINKED LIST AS QUEUE

Doubly linked linear lists can be easily used to represent a queue whose number of elements is very volatile. Such representation can see as:



Doubly linked linear list representation of a queue

Here ***R*** and ***F*** are pointer variables which denote the ***REAR*** and ***FRONT*** of the queue, respectively. The insertion of a node whose address is ***NEW*** at the ***REAR*** of the queue as shown below:



Insertion in a doubly linked queue

The following sequence of steps accomplishes such insertion:

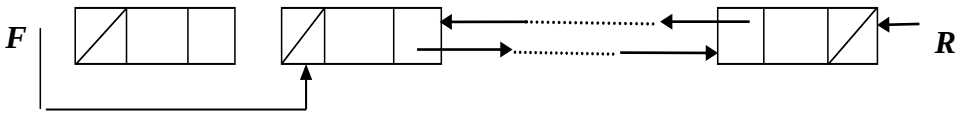
RPTR (R) = NEW;

RPTR (NEW) = NULL;

LPTR (NEW) = R;

R = NEW;

Similarly, a deletion from a doubly linked queue can represent as:



Deletion in a doubly linked queue

The deletion from the front of the queue is achieved by the following algorithmic steps:

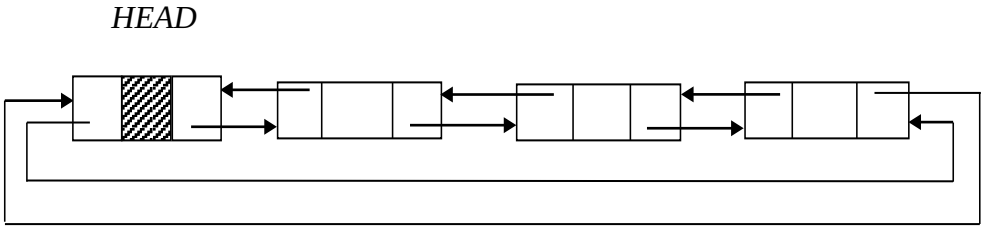
F = RPTR (F);

LPTR (F) = NULL;

7.14 CIRCULARLY DOUBLY LINKED LIST

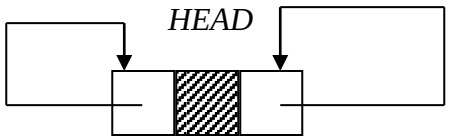
The process of insertion and deletion can simplify if we combine the advantages of doubly linked list and circular header linked list. The doubly linked list may implement as the circular linked list by connecting the two end nodes point back to its header node. Therefore the case of an empty list is dispensed with by never permitting a list to be empty. This can be accomplished by using a special node that always remains in the list. Hence, it is the only node in an empty list. The special node is called the *Head node* of the list. Thus, such a two-way list requires only one list

pointer variable i.e. *START*, which points to the header node. This is because the two pointers in the header node point to the two ends of the list as shown in the following graphical representation of the circular doubly linked list:



A doubly linked circular list with a head node

In this representation we can see that the right link of the right-most node contains the address of the head node and the left link of the head node points to the right-most node. The empty list can present when both left and right links of the head node point to itself. This can view graphically as:



An empty doubly linked circular list with a head node.

The algorithm for inserting a node in the doubly linked circular list to the left of a specified node ***M*** now reduces to the following sequence of steps with respect to the insertion algorithm in a double linked linear list:

Algorithm *DOUBCRINS (L, R, M, X):*

[Given a doubly circular linked list with the ***HEAD*** node. Now it is required to insert a node whose address is given by the pointer variable ***NEW***. The left and right links of a node are denoted by ***LPTR*** and ***RPTR***, respectively. The information field of a node is denoted by the variable ***INFO.*** The name of an element of the list is ***NODE***. The insertion is to be performed to the left of a specified node with its address given by the pointer variable ***M***. The information to be entered in the node is contained in ***X***.]

1. *[obtain new node from availability stack]*

   NEW ⇐NODE;

2. *[copy information field]*

   INFO (NEW) = X;

3. *[Insertion the node]*

   RPTR (NEW) = M

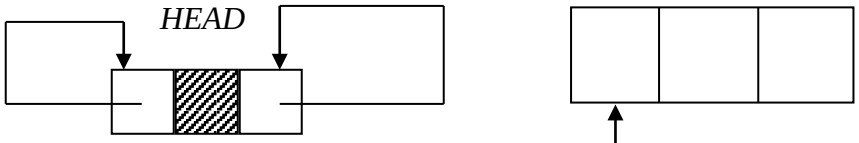
LPTR (NEW) = LPTR (M)

RPTR (LPTR (M)) = NEW

LPTR (M) = NEW

Return (NEW);

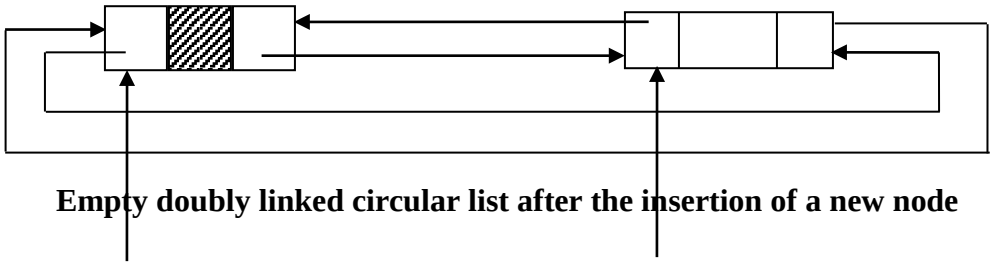
The insertion of a node into an empty list can be represented before and after the insertion as follows:



Empty doubly linked circular list

Before the insertion of new node **NEW**

HEAD



Empty doubly linked circular list after the insertion of a new node

M **NEW**

In the same manner, the deletion algorithm of a node with an address given by the variable ***OLD*** from the doubly linear linked is modified and this modification can present in the following steps as:

Algorithm *DOUBCRDEL (L, R, OLD) :*

[Given a doubly linked circular list with the ***HEAD*** node**.** This is required to delete the node whose address is given by the pointer variable ***OLD***. The left and right links of a node are denoted by ***LPTR*** and ***RPTR***, respectively.]

1. *[check the condition for underflow]*

   If (HEAD (LPTR) == HEAD (RPTR))

   {

   Printf (”Underflow”);

Return

}

2. *[Delete the node]*

   {

   RPTR (LPTR (OLD)) = RPTR (OLD);

   LPTR (RPTR (OLD)) = LPTR (OLD);

   }

3. *[Return deleted node]*

   Return (OLD);

7.15 APPLICATION OF LINKED LIST: POLYNOMIAL REPRESENTATION

Here we consider the applications of linked list. A very common and important application of linked list is for the representation of a polynomial. A polynomial, p (x), is an expression in variable x of the form ($ax^n + bx^{n-1} + ... + jx + k$) where a, b, c, …k are real numbers and n is a non-negative integer. The number *n* is called the degree of the polynomial. An important characteristic of a polynomial is that each term in the polynomial expression consists of following two parts:

- **Coefficient**
- **Exponent**

Consider the following polynomial:

$$ax^5 + bx^3 - cx^2 - dx$$

Here, (a, b, -c, -d) are coefficients and (5, 3, 2, 1) are exponents.

Exponents are the placeholders for any value that remains constant for each term in a single expression. In data structure, a polynomial can be represented as a list of nodes where each node consists of coefficient and an exponent.

Points to be considered when working with polynomials are:

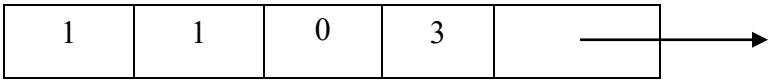
- Sign of each coefficient and exponent is stored within the coefficient and exponent itself.
- Only addition of term with equal exponent is possible.
- Storage of each term in the polynomial must be done in ascending / descending order of their exponent.

Consider the example of representing a term of a polynomial in the variables x, y, z. A typical node of the linked list can represent as:

POWER_X	POWER_Y	POWER_Z	COEFF	LINK

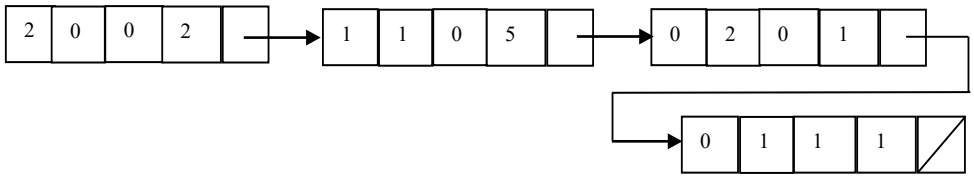
A typical node of the linked list for the representation of a polynomial of three variables

This typical node consists of five sequentially allocated fields that we collectively refer to as ***TERM***. The first three fields represent the power of the variables x, y, z, respectively. The fourth and fifth fields represent the coefficient of the term in the polynomial and the address of the next term in the polynomial, respectively. For example, the term ***3xy*** would be represented as:



We consider the pointer address ***P*** to reference the node for our algorithmic notation. *COEFF (P)* denotes the coefficient field of a node pointed to by *P*. Similarly, the exponents of x, y and z are given by *POWER_X (P)*, *POWER_Y (P)*, and *POWER_Z (P)*, respectively, and the pointer to the next node is given by *LINK (P)*.

Consider as an example the representation of the polynomial: $2x^2 + 5xy + y^2 + yz$ as a linked list. Assume that the nodes in the list are to be stored such that a term pointed to by *P* precedes another term indicated by *Q* if *POWER_X (P)* is greater than *POWER_X (Q)*; or, if the power of x is equal, then *POWER_Y (P)* must be greater than *POWER_Z (Q)*. For our example, the list is represented as:



We can formulate an algorithm which inserts a term of a polynomial into a linked list.

Algorithm *POLYFRONT (NX, NY, NZ, NCOEFF, POLY):*

[Given the definition of the node structure ***TERM*** and an availability area from which we can obtain such nodes, it is required to insert a node in the linked list so that it immediately precedes the node whose address is designated by the pointer ***POLY***. The fields of the new term are denoted by ***NX, NY, NZ,*** and ***NCOEFF***, which correspond to the exponents for x,

y, and z, and the coefficient value of the term, respectively. ***NEW*** is a pointer variable which contains the address of the new node.]

1. *[Obtain a node from available storage]*

   NEW ⇐ TERM

2. *[Initialize numeric fields]*

   POWER_X (NEW) = NX;

   POWER_Y (NEW) = NY;

   POWER_Z (NEW) = NZ;

   COEFF (NEW) = NCOEFF

3. *[Set link to the list]*

   LINK (NEW) = POLY;

4. *[Return first node pointer]*

   Return (NEW)

This function performs all its insertions at one end of the linked list. In general, it is also possible to perform insertions at the other end or in the middle of the list. The zero polynomial (polynomial with no terms) is represented by the *NULL* pointer. Before any term of a polynomial has been added to a list, its first node pointer, which we call *POLY*, has a value of *NULL*. When function *POLYFRONT* is invoked, the address of the created node is returned, and it is this value that replaces the function call i.e. *POLY = POLYFRONT (NX, NY, NZ, NCOEFF, POLY*. The construction of a linked list for a polynomial is achieved by having a zero polynomial initially and by repeatedly invoking function *POLYFRONT* until all terms of the polynomial are processed.

7.16 STACK IMPLEMENTATION WITH LINKED LIST

The problem with array-based stacks is that the size must be determined at compile time. Thus the size of the stack is fixed. The stack implementation with array reflects the static memory allocation. Hence to implement the stack with dynamic memory allocation we use a linked list, with the stack pointer pointing to the *TOP* element, let *FRESH* be the new node. To push a new element on the stack, we must do:

FRESH->NEXT= TOP;

TOP = FRESH;

To pop an item from a linked stack, we just have to reverse the operation.

ITEM = TOP;

TOP = TOP->NEXT;

7.17 GARBAGE COLLECTION

Garbage collection is the process of collecting all unused nodes and returning them to available space. Therefore the garbage collection is about freeing dynamically allocated memory when not in use. This process is carried out in two phases:

- First phase, known as marking phase. It involves marking of all nodes that are accessible from the external pointer.
- Second phase, known as collection phase. It involves proceeding sequentially through memory and freeing all nodes that have not been marked.

In the implementation for linked list when we delete a node from the linked list and return it to the memory back i.e. *free (node)* then the space allocated to the deleted node and its pointer address return back to the memory. This process is considered as the garbage collection.

7.18 SUMMARY

Linked list is a linear dynamic data structure which allows storing the elements in non-contiguous memory locations. It is not restricted as the array. It provides the way of dynamic memory allocation. The contents of this unit can be summarized as:

- Linked list is the most commonly used data structure to store similar type of data in memory.
- It is linear type data structure but allocate the memory in non contiguous manner.
- It is a data structure which allocates the dynamic memory allocation aspect.
- Self referential structure and pointer data types may represent the singly connected linked lists.
- To make the traversal operation easy, doubly connected linked lists are used, in which every node contains links to its left and right neighbours.
- The *NULL* value in the end of a single linked list denotes the end of the list. The *NULL* link when set to the beginning of the list, results in the list called circular linked list.
- The idea of dynamic memory allocation is to be able to allocate and de-allocate memory at runtime in response to program requirement and thus manage that space efficiently.
- Stack and queue can implement with the linked linear lists.

- The polynomial can represent and implement with linked list.

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SELF EVALUATION

Multiple Choice Questions:

1. In linked list, a node contains:
   a. Node, address field and data field.
   b. Node number, data field
   c. Next address field, information field.
   d. None of these
2. In linked list, the logical order of elements
   a. Is same as their physical arrangement
   b. Is not necessarily equivalent to their physical arrangement
   c. Is determined by their physical arrangement.
   d. None of the above
3. *NULL* pointer is used to tell
   a. End of linked list.
   b. Empty pointer field of a structure
   c. The linked list is empty
   d. All of the above
4. List pointer variable in linked list contains address of the:
   a. Following node in the list
   b. Current node in the list
   c. First node in the list
   d. None of the above
5. Due to the linear structure of linked list having linear ordering, there is similarity between linked list and array in:
   a. Insertion of a node
   b. Deletion of a node
   c. Traversal of element of list
   d. None of the above
6. Searching of linked list requires linked list to be created:
   a. In stored order only.
   b. In any order

c. Without underflow condition

d. None of the above

Fill in the blanks

1. The next address field is known as..... (pointer / address)
2. Linked list provides ..... memory allocation (static / dynamic)
3. In linked list, the id entity of next element is.....defined. (explicitly / implicitly)
4. Beside data field, each node of linked list contains at least..... more fields. (one / two)
5. End of the linked list is marked by putting..... in the next address field in the last node. (next / *NULL* pointer)
6. Attempting to delete a new node in ..... linked list results in underflow. (Empty / non-empty).
7. Polynomials in memory can be represented by ..... lists. (Linear / Circular).
8. For representing polynomial in memory using linked list each node must have ..... fields (three / two)
9. A polynomial is made of different terms each of which consists of a ..... and .....

State whether True or False

1. Doubly linked list is the two way linked lists.
2. List – null can be used to initialize list as empty list.
3. In linked list, successive elements need not occupy adjacent space in memory.
4. Circular linked list can be used without *header* node efficiently.
5. Linear queue cannot implement with doubly linked list.

Answer the following questions

1. Write a program in "C" which reads the name, age and salary of 10 persons and maintains them in linked list sorted by name.
2. There are two linked lists A and B containing the following data:

   A : 2, 5, 9, 14, 15, 7, 20, 17, 30

   B : 14, 2, 9, 13, 37, 8, 7, 28

   Write programs to create :

   (i) A linked list C that contains only those elements those are common in linked list A and B.

(ii) A linked list D which contains all elements of A as well as B ensuring that there is no repetition of elements.

3. Define polynomial as an abstract data type. Write a "C" functions to add the two polynomials and return the sum.

4. What do you mean by linked list? What are the elements available in the list? Specify the advantages of doubly linked list over the singly linked list. What is garbage collection?

5. Write a function in "C' that constructs a doubly linked list with a list head from a singly linked list that is accessed through a pointer *FIRST*. The original singly linked list need not be destroyed.

6. Write a program in "C" to insert and delete the element from the circular doubly linked list.

UNIT-8 TREE

Structure

8.0 INTRODUCTION

This unit introduces one of the most important data structures which is of type nonlinear and noncontiguous. The tree is a fundamental structure in computer science. Almost all operating systems store files in tree or tree like structures. Trees are also used in complier design, text processing and searching algorithms. In this unit the concept of tree is introduced with various kinds of trees and the operations on these trees. This unit starts with the introduction of basic terminology used for the tree. It introduces the concept of binary tree, complete binary tree and extended binary tree and their representation with array and linked list. This unit gives a detailed account of the various operations that can be performed on the binary tree like traversing and searching in the binary tree. The concept of threaded binary tree is introduced with its implementation. This unit covers the discussion about BST, AVL tree and

B-tree with the operations of insertion and deletion of a node in these trees. Various examples are used to elaborate the operations of traversing, insertion and deletion from the tree.

8.1 OBJECTIVES

After going through this unit, you should be able to:

- Define and understand the basic terminology used for tree.
- Understand the concept of tree, binary tree, complete binary tree and extended binary tree.
- Explore of various operations like traversing, searching, insertion and deletion from the binary tree.
- Understand and implementation of the binary search tree.
- Understand the examples for showing these operations in the binary tree.
- Understand and implementation of the threaded binary tree.
- Define and understand the BST, AVL tree and B-tree
- Understand the concept of these trees with their examples.
- Implementation of insertion and deletion operations in the B- Tree.

8.2 TREE AND ITS BASIC TERMINOLOGY

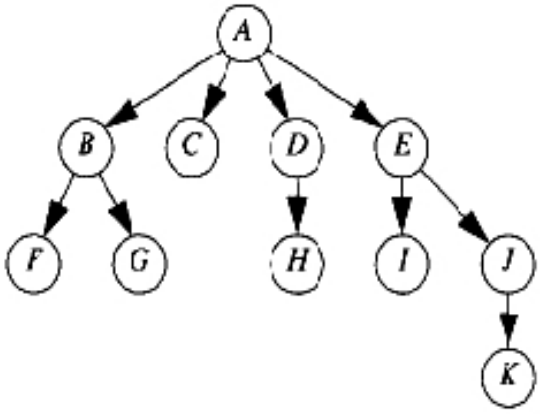
A ***Tree*** consists of a set of nodes and a set of directed edges that connect pairs of nodes. ***Trees*** are useful in describing any structure which involves hierarchy. Familiar examples of such structures are family tree, the decimal classification of books in library, the hierarchy of positions in an organization, an algebraic expression involving operations for which certain rules of precedence are prescribed. A ***Tree*** consists of a main node from where all the branches emerged i.e. top of the tree which is called as the ***Root*** node of the ***Tree***. A rooted ***Tree*** has the following properties:

- One node is distinguished as the root.
- Every node *c,* expect the root, is connected by an directed edge from exactly one other node *p.* Node *p* is *c*'s ***parent***, and *c* is one of *p*'s ***children***.
- A unique path traverses from the root to each node. The number of edges that must be followed is the ***path length.***

The number of edges emerging from a node is called the *out degree* of the node. Thus, in a directed tree, any node which has *out degree* zero is called a ***terminal node*** or ***leaf node.*** Other nodes those have nonzero out degree are called the ***branch nodes***. The number of edges directing for a

node is called the *in degree* of the node. Thus, in a directed tree, any node which has *in degree* zero is called a ***root node.*** It is important to note that every tree must have at least one node. A single isolated node is also called as the directed tree.

The ***level*** of any node is the length of its path from the root. The level of the root of a directed tree is zero, while the level of any node is equal to its distance from the root.



Node	Height	Depth
A	3	0
B	1	1
C	0	1
D	1	1
E	2	1
F	0	2
G	0	2
H	0	2
I	0	2
J	1	2
K	0	3

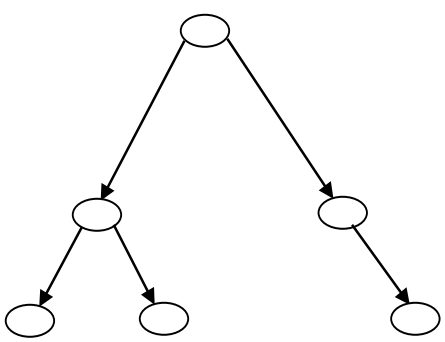
The Tree with the information of height and depth

In this given tree the root node is *A*: *A's* children are *B, C, D* and *E*. Because *A* is the root, it has no parent. All other nodes have parents like *B's* parent is *A*. The leaves in this tree are *C, F, G, H, I and K.* The length of the path from *A* to *K* is 3 (edges). The length of the path from *A* to *A* is 0 (edges). A tree with *N* nodes must have *N-1* edges because every node except the parent has an incoming edge. The ***depth of a node*** is a tree is the length of the path from the node to the deepest leaf. Thus the height of *E* is 2. The height of any node is 1 more than the height of its maximum height child. Thus, the height of a tree is the height of the root.

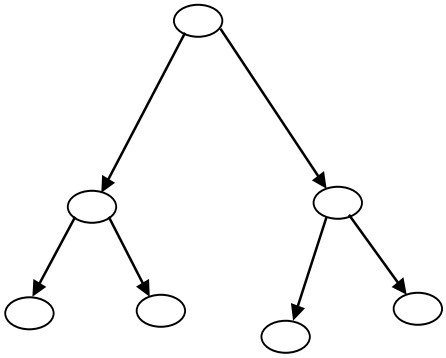
Nodes with the same parent are called ***siblings***. Thus *B, C, D* and *E* are ***siblings***. If there is a path from any node *u* to node *v*, then *u* is an ***ancestor*** of *v* and *v* is a ***descendant*** of *u*. If $u \neq v$, then *u* is a ***proper ancestor*** of *v* and *v* is a ***proper descendant*** of *u.* The ***size of a node*** is the number of descendants the node has (including the node itself). Thus the size of *B* is 3, and the size of *C* is 1. The size of a tree is the size of the root. Thus the size of the given tree is the size of its root *A*, or 11.

RIL-102 If in a directed tree the out degree of every node is less than or equal to *m*, then the tree is called an *m-ary tree.* If the out degree of every node is

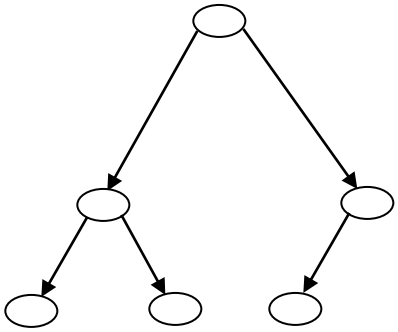
exactly equal to m or 0 and the number of nodes at level i is m^{i-1} (assuming that the root node has a level number of 1), then the tree is called a *full* or *complete m- ary tree*. For $m = 2$, the trees are called ***binary and complete binary tree.*** We shall now *m- ary tree* consider in which the m (or fewer) children of any node are assumed to have m distinct positions. If such positions are taken into account, then the tree is called a *positional m- ary tree*. The following graphical representations are expressing the different form of the *binary tree*.



A Binary Tree



A Complete Binary Tree



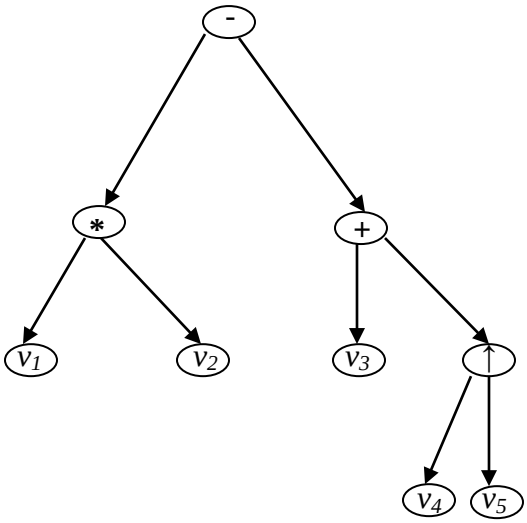
A Binary Tree with distinct position

Example :

Represent the following expression with binary tree.

$v_1 * v_2 - (v_3 + v_4 \uparrow v_5)$

The binary tree representation is as follows:



Therefore we can define the ***Tree*** formally as:

A *tree* is a non linear data structure and is generally defined as a non empty finite set of elements, called nodes such that:

- *Tree* contains a distinguished node called *root* of the tree.
- The remaining elements of the tree form an ordered collection of zero or more disjoint subsets called *sub tree.*

8.3 BINARY TREE

A ***Binary tree*** is a special type of tree in which every node or vertex has no children, one child or two children. A ***Binary tree*** is an important class of tree data structure in which a node can have at most ***two*** children (as sub trees). Child of a node in a binary tree on the left is called the "***left child***" and the node in the right is called the "***right child***". A binary tree is defined as a finite set of elements, called nodes, such that:

- Tree is empty (called the null tree of empty tree) or
- Tree contains a distinguished node called root node, and the remaining nodes form an ordered pair of disjoint binary trees.

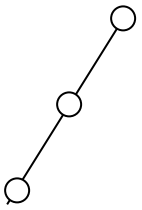
Complete Binary Tree

A binary tree is said to be complete if all its level except the last, have maximum number of possible nodes, and if all the nodes at the last level appear as far left as possible as shown below:

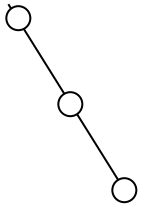
- If a tree has *n* nodes then the number of branches it has is *n-1*.
- Every node in a tree has exactly one parent except the root node.
- Extended binary tree can have either no children or two children.
- For binary tree of height ***h*** the maximum number of nodes can be $2^{h+1}-1$.
- Any binary tree with *n* internal nodes has *(n+1)* external nodes.

Skewed Tree

A tree is called Skew if all the nodes of a tree are attached to one side only. i.e A left skew will not have any right children in its each node and right skew will not have any left child in its each node.



Left Skew



Right Skew

Heap

A binary tree is also called a heap and there are two types of heap. There are Max Heap and Min Heap. A heap is called maximum heap if value of a node is greater than or equal to each of its descendant node. A heap is called minimum heap if value of a node is less than or equal to each of its descendant node.

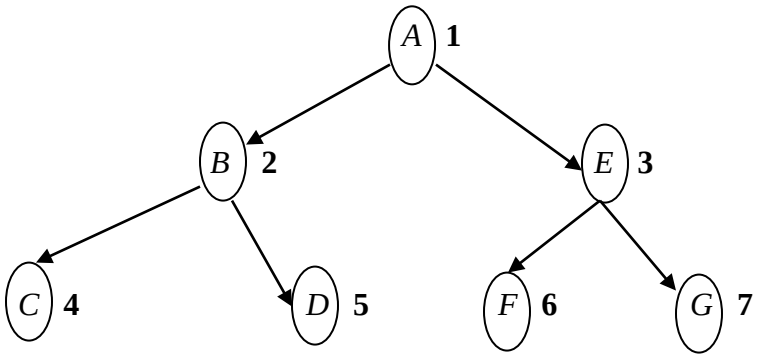
8.4 BINARY TREE REPRESENTATION

Any binary tree can be represented in two ways. The first way is with sequential manner like array and second way is with linked list.

The ***sequential representation*** of tree stores data in an array as per the following rules:

1. The root node is stored in 1st position.
2. Every left and right child of a parent node at location k will be stored in $(2*K)^{th}$ position and $(2*K+1)^{th}$ position respectively.

An example of such a tree structure, together with its sequential representation can show as follows:



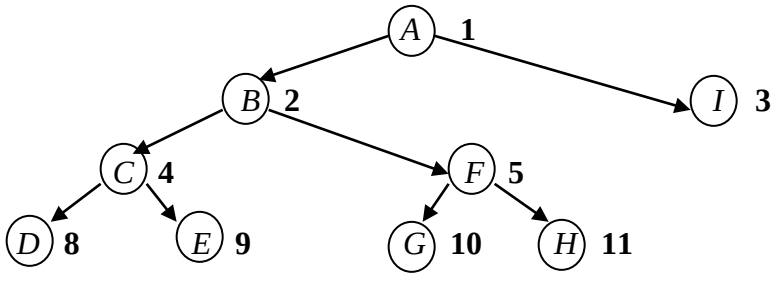
The array representation of this tree is expressed as:

Position	1	2	3	4	5	6	7
INFO	*A*	*B*	*E*	*C*	*D*	*F*	*G*

Sequential representation of a complete binary tree

In this representation the locations of the left and right children of node *i* are *2i and 2i +1,* respectively. For example the index of the left child of the node in position 3 (that is *E*) is 6. Similarly the index of the right child is 7. Conversely, the position of the parent of node *j* is the index *int (j / 2).* For example the parent of node 4 and 5 is 2.

Now we consider another example for representing the incomplete binary tree with sequential representation.



The array representation of this tree is expressed as:

Position	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
INFO	*A*	*B*	*I*	*C*	*F*	-	-	*D*	*E*	*G*	*H*	-	-	-	-

Sequential representation of an incomplete binary tree

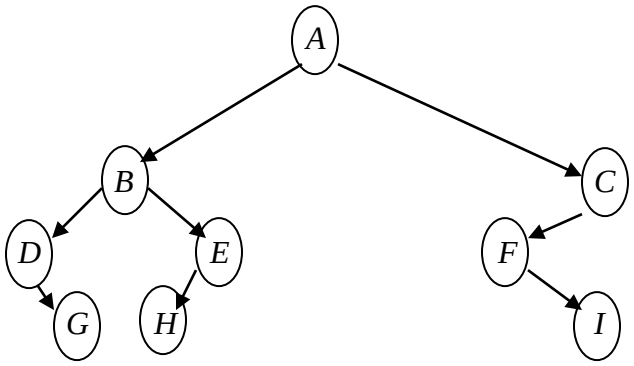
We can observe from here is that a substantial amount of memory is wasted in this case. Therefore, for large trees of this type, this method of representation may not be efficient in terms of storage.

8.5 LINKED STORAGE REPRESENTATION FOR BINARY TREES

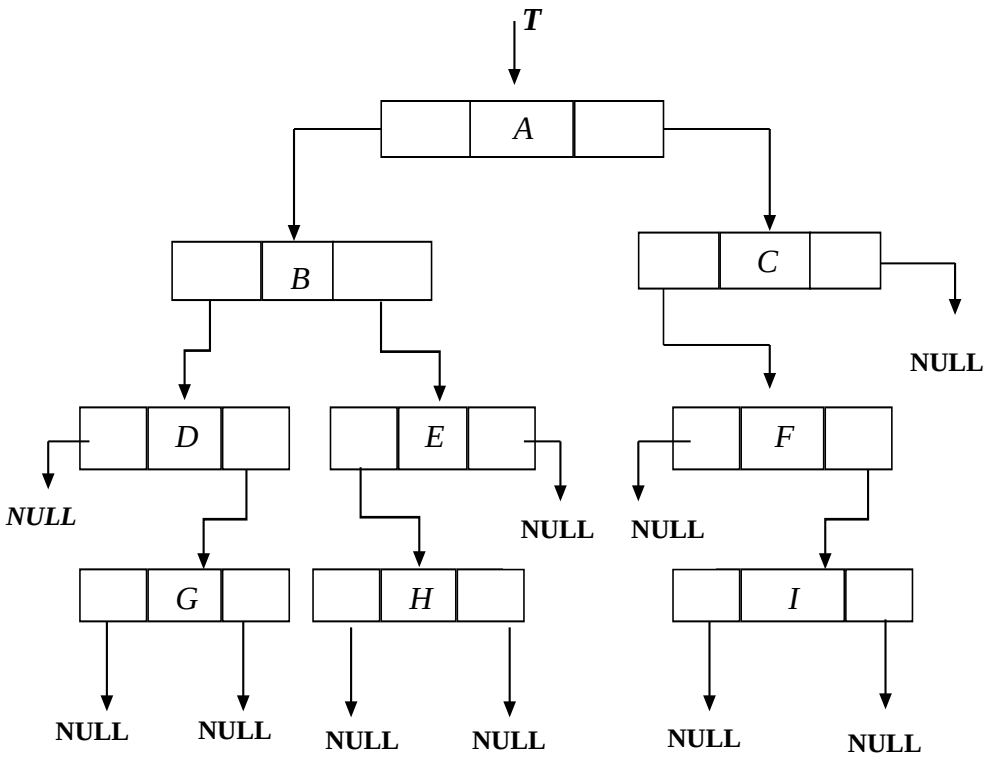
Since a binary tree consists of nodes which can have at most two offspring, an obvious linked representation of such a tree involves having storage nodes as follows:

LPTR	*DATA*	*RPTR*

Here *LPTR* and *RPTR* denote the addresses or locations of the left and right sub-trees, respectively, of a particular root node. Empty sub-trees are represented by a pointer value of *NULL*. *DATA* specifies the information associated with a node. Let us consider an example for the binary tree and its representation with linked list form. Now consider the following binary tree:



The linked list representation of this binary tree is:



The pointer variable ***T*** denotes the address of the root node.

8.6 TRAVERSING BINARY TREE

One of the most common operations performed on tree structures is that of traversal. Traversing is the process of visiting every node in the tree exactly once in a systematic manner. Therefore, a complete traversal of binary tree implies visiting or processes the nodes of the tree in some linear sequence. If a particular sub tree is empty then the traversal is performed by doing nothing. Thus, a null sub-tree is considered to be fully traversed when it is encountered.

For example, a tree could represent an arithmetic expression. In this context the processing of a node which represents an arithmetic operation would probably mean performing or executing that operation. There are three standard ways of visiting a binary tree ***T*** with root ***R***:

- Preorder or depth-first order
- In-order or symmetric order
- Post-order

The easiest way to define the order of traversing is with recursion. So we define these orders of traversing recursively:

The *Pre-order traversal*

The *pre-order* of a binary tree is defined as follows:

1. Process the root node ***R***.
2. Traverse the left sub-tree in *pre-order* (Recursive call)
3. Traverse the Right Sub tree in *Pre-order* (Recursive call)

The *In-order traversal*

The *in-order* of a binary tree is given by the following steps:

1. Traverse the left sub-tree in *In-order* (Recursive call)
2. Process the root node ***R***.
3. Traverse the Right Sub tree in *In-order* (Recursive call)

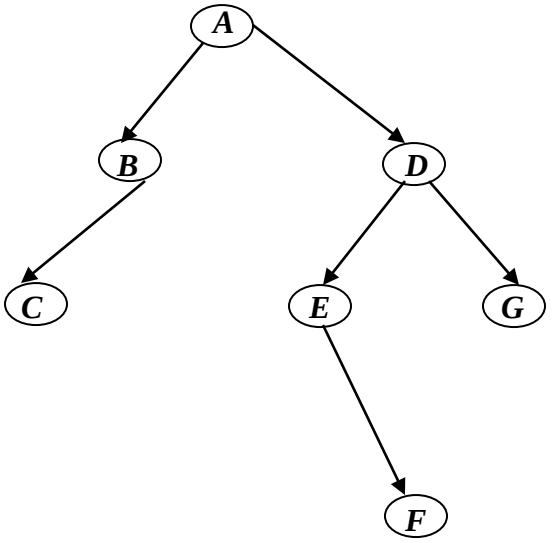
The *Post-order traversal*

The *Post-order* of a binary tree is given by the following steps:

1. Traverse the left sub-tree in *Post-order* (Recursive call)
2. Traverse the Right Sub tree in *Post-order* (Recursive call)
3. Process the root node ***R***.

Example

Now we consider an example of a binary tree and its traversal from all three orders i.e. *Pre-Order, In-Order and Post-Order*.



The *Pre-Order travel* sequence is as follows:

A B C D E F G

The *In-Order travel* sequence is as follows:

C B A E F D G

The *Post-Order travel* sequence is as follows:

C B F E G D A

Now we can formulate the algorithms for the traversal. These algorithms can formulate in recursive as well as in iterative manner.

Let us consider the traversal of binary trees by **iterations**. Since in traversing a tree it is required to descend and subsequently ascend parts of the tree, pointer information which will permit movement up the tree must be temporarily stored. Because movement up the tree must be made in a reverse manner from that taken in descending a tree, a ***stack*** is required to save pointer variables as the tree is traversed. A general algorithm for a *Pre-order* traversal of a binary tree using iteration is given as:

Algorithm *PREORDER (T):* **[**Given a binary tree whose root node address is given by a pointer variable ***T.*** This method traverses the tree in *Pre-Order* in iterative manner. ***S*** and ***TOP*** denote an auxiliary stack and its associated top index, respectively. The pointer variable ***P*** denotes the current node in the tree.]

/ Initialize the pointers */*

If (T == NULL)

{

Printf ("Empty tree")

Return

}

Else {

TOP = 0;

PUSH (S, TOP, T) / the PUSH is the function call which is already defined in stack previous unit */*

}

While (TOP > 0)

{

P = POP (S, TOP) / get stored address and branch left. The PUSH is the function call which is already defined in stack previous unit */*

While (P != NULL)

Printf (DATA (P));

If (RPTR (P) != NULL)

PUSH (S, TOP, RPTR (P)); / store address of non-empty right sub-tree */*

P = LPTR (P)

}

Return;

A trace of the algorithm for the given binary tree as described above already in the example can show as in the following table:

Stack	*Comments*	*P*	*Visit P*	*Output String*
A				
		A	*A*	*A*
D		*B*	*B*	*AB*
D		*C*	*C*	*ABC*
D		*NULL*		
		D	*D*	*ABCD*
G		*E*	*E*	*ABCDE*
G F		*NULL*		
G		*F*	*F*	*ABCDEF*
G		*NULL*		
		G	*G*	*ABCDEFG*
		NULL		

Trace of algorithm PREORDER for the given binary tree in the example

An equivalent algorithm for a *Pre-order* traversal of a binary tree using recursion can formulate as:

Algorithm *RPREORDER (T):* **[**Given a binary tree whose root node address is given by a pointer variable ***T***. This algorithm traverses the tree in *Pre-order* in a recursive manner.]

1. */* Process the root node */*

   If (T != NULL)

   Printf (DATA (T));

Else {

Printf ("EMPTY TREE");

Return

}

2. *If (LPTR (T) != NULL)*

   RPREORDER (LPTR (T)); / process the left sub-tree */*

3. *If (RPTR (T) != NULL)*

   RPREORDER (RPTR (T)); / process the right sub-tree */*

4. *Return*

A general algorithm for a *Post-order* traversal of a binary tree using iteration is given as:

Algorithm *POSTORDER (T):* **[**Given a binary tree whose root node address is given by a pointer variable ***T.*** This method traverses the tree in *Post-Order* in iterative manner. ***S*** and ***TOP*** denote an auxiliary stack and its associated top index, respectively. The pointer variable ***P*** denotes the current node in the tree. In this process each node will be stacked twice, once when its left sub tree is traversed and once when it right sub tree is traversed. On completion of these two traversals, the particular node is processed. Consequently, we need two types of stack entries, the first indicating that a left sub-tree is being traversed, and the second that a right sub-tree is being traversed. For convenience we use negative pointer values to indicate the second type of entry.]

1. */* Initialize the pointers */*

   If (T == NULL)

   {

   printf ("Empty TREE");

   Return;

   Else {

   P = T;

   Top = 0;

   }

2. *[Traverse in Post-order]*

   Repeat thru step 5 while true

3. *[Descend left]*

   While (P != NULL) {

   PUSH (S, TOP, P);

P = LPTR (P); }

4. *[Process a node whose left and right sub-tree have been traversed]*

   While (S [TOP] < 0)

   {

   P = POP (S, TOP);

   Printf (DATA (P));

   If (TOP == 0)

   Return;

   }

5. *[Branch right and then mark node from which we branched]*

   P = RPTR (S [TOP]);

   S [TOP] = -S [TOP];

An equivalent algorithm for a *Post-order* traversal of a binary tree using recursion can formulate as:

Algorithm *RPOSTORDER (T):* **[**Given a binary tree whose root node address is given by a pointer variable ***T***. This algorithm traverses the tree in *Pre-order* in a recursive manner.]

1. */* Check for empty tree */*

   If (T == NULL)

   {

   Printf ("EMPTY TREE");

   Return

   }

2. *[Process the left subtree]*

   If (LPTR (T) != NULL)

   RPOSTORDER (LPTR (T)); / process the left sub-tree */*

3. *If (RPTR (T) != NULL)*

   RPOSTORDER (RPTR (T)); / process the right sub-tree */*

4. *[Process the root node]*

   Printf (DATA (T));

5. *[Finished]*

   Return

Now we present the recursive algorithm for traversing the binary tree in *In-order*.

Algorithm *RINORDER (T):* **[**Given a binary tree whose root node address is given by a pointer variable ***T***. This algorithm traverses the tree in *Pre-order* in a recursive manner.]

1. *If (T == NULL)*

   {

   Printf ("EMPTY TREE");

   Return

   }

2. *[Process the left subtree]*

   If (LPTR (T) != NULL)

   RINORDER (LPTR (T)); */* process the left sub-tree */*

3. *[Process the root node]*

   Printf (DATA (T));

4. *If (RPTR (T) != NULL)*

   RINORDER (RPTR (T)); */* process the right sub-tree */*

5. *[Finished]*

   Return

8.7 OPERATION ON THE BINARY TREE

Now we discuss the various operations on the binary tree. The traversing operation we have discussed already. There are some other important operations also exist for the binary tree. These operations are:

- Copy of the tree
- Insertion of an element in the tree
- Deletion of an element from the tree
- Reconstruction of binary tree

Hence we discuss these operations with the description of algorithm one by one.

Copy of the Tree

The copy of the binary tree is an important operation for the tree. It provides a duplicate copy of the binary tree. The original tree may be destroyed during the manipulation or processing. So that it is required to maintain a copy of the binary tree. Therefore, a copy of the tree is

produces before such processing begins. The following algorithm generates a copy of a tree.

Algorithm *Copy (T)* [Given a binary tree whose root node address is given by the pointer ***T***. This algorithm generates a copy of the tree and returns the address of its root node. ***New*** is a temporary pointer variable.]

1. *[Check for the NULL pointer]*

   If (T ==NULL)

   Return (NULL);

2. *[Create a new node]*

   NEW ⇐NODE;

3. *[Copy of the information field]*

   DATA (NEW) = DATA (T);

4. *[Set the structural links]*

   LPTR (NEW) = COPY (LPTR (T));

   RPTR (NEW) = COPY (RPTR (T));

5. *[Return address of new node]*

   Return (NEW);

Insertion in the tree

A tree can be created through the repeated use of an insertion operation. Now we assume that a binary tree exists. Such a tree, however, must be constructed. This construction can be realized by the repeated use of an insertion operation that adds a new node into an existing tree. For example the insertion of a node into a lexically ordered tree must maintain that ordering. Such an insertion is performed at the leaf level. There are two cases arise:

1. As a special case, an insertion into an empty tree results in appending the new node as the root of the tree.

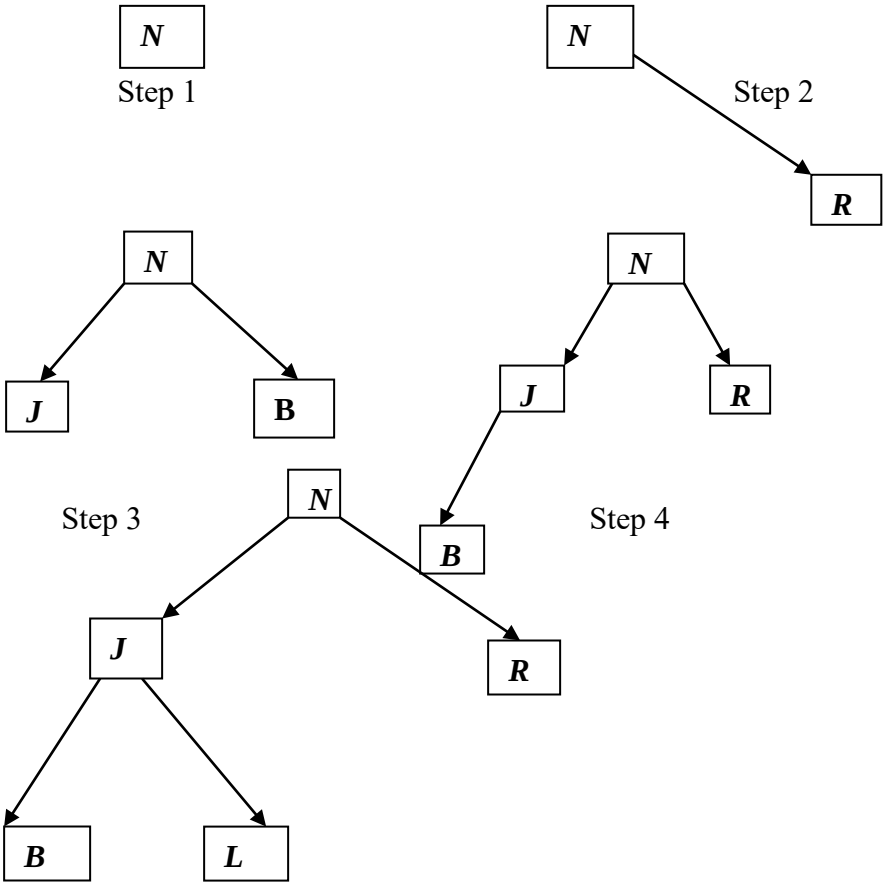
2. The more general case involves inserting a new node into a nonempty tree. The new node i.e. in lexically ordered tree the new name is first compared with the name of the root node. If the new name lexically precedes the root node, then the new node is appended to the tree as a left leaf to the existing tree if the left sub tree is empty, otherwise the comparison process is repeated with the root node of the left sub tree. If on the other hand, the new name lexically follows the root node name, then the new node is appended as a right leaf to the present tree if the right sub-tree is empty, otherwise the comparison process is repeated with the root node of the right sub-tree.

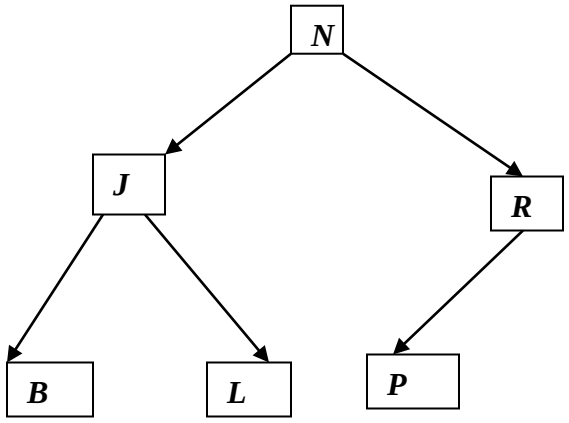
Example

Let us consider the following alphabets for the insertion in the tree:

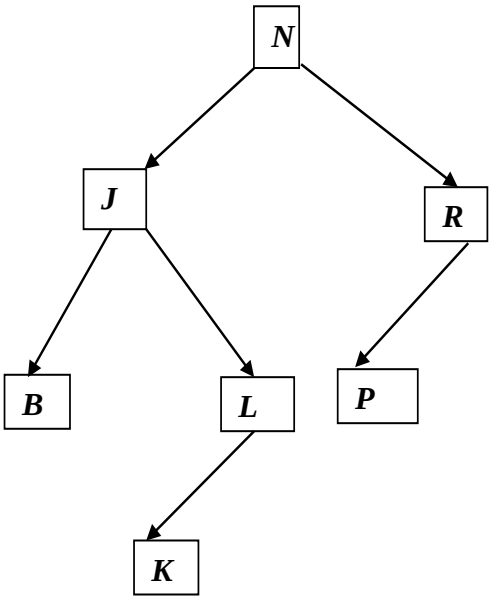
N, R, J, B, L, P, K and M

A trace of the construction or insertion of the tree can exhibit as:

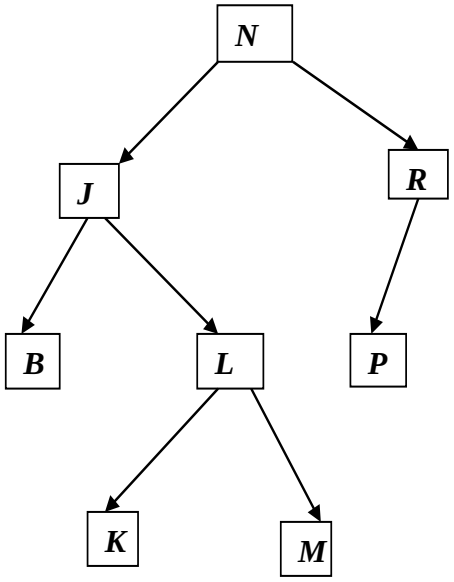




Step 5



Step 6



Step 7

As we have seen from the example that a tree can be created through the repeated use of an insertion operation. A general algorithm for performing such a n i nsertion i nto an e xisting l exically o rdered bi nary t ree i s a s follows:

1. *If the existing tree contains no nodes then append the new node as the root of the tree and exit.*
2. *Compare the new name with the name of the root node,*

   If the new name is lexically less than the root node name

   Then if the left sub tree is not empty

   Then repeat step 2 on the left sub-tree

   Else append the new name as a left leaf to the present tree

   Exit

   Else if the right sub-tree is not empty

   Then repeat step 2 on the right sub tree

   Else append the new name as a right leaf to the present tree

   Exit.

Deletion from the tree

Now we have the inverse problem of insertion i.e. the deletion. Here we are defining the algorithm for deleting i.e. removing the node with key ***X*** in a tree with ordered keys. This process is straight forward if the element to b e d eleted i s a t erminal n ode or one w ith single de scendant. T he difficulty a rises i n r emoving a n e lement w ith t wo de scendants, f or w e

cannot point in two directions with a single pointer. Here in this situation the deleted element is to be replaced by either the rightmost element of its left sub-tree or by the leftmost node of its right sub tree, both of which have at most one descendant. This procedure distinguishes among three cases:

1. There is no component with a key equal to ***X***.
2. The component with key ***X*** has at most one descendant.
3. The component with key ***X*** has at most two descendants.

The detail process of delete in recursive manner can represent as follows:

Algorithm *TREE_DELETE (HEAD, X)* [Given a lexically ordered binary tree with the node structure previously described and the information value ***(X)*** of the node marked for deletion, this procedure deletes the node whose information field is equal to ***X***. ***PARAENT*** is a pointer variable which denotes the address of the parent of the node marked for deletion. ***CUR*** denotes the address of the node to be deleted. ***PRED*** and ***SUC*** are pointer variables used to find the in-order successor of ***CUR***. ***Q*** contains the address of the node to which either the left or right link of the parent of ***X*** must be assigned in order to compete the deletion. Finally, ***D*** contains the direction from the parent node to the node marked for deletion. It is assumed that the tree have a list head whose address is given by ***HEAD***. ***FOUND*** is a Boolean variable which indicates whether the node marked for deletion has been found. '***L***' and ***'R'*** are representing the left branch and right branch respectively.]

1. *[Initialize]*

   If LPTR (HEAD) !=HEAD

   {

   CUR = LPTR (HEAD);

   PARENT = HEAD;

   D = 'L'

   }

   Else

   {

   Printf ('NODE NOT FOUND')

   Return;

   }

2. *[Search for the node marked for deletion]*

```
FOUND = false;
While ((! FOUND) && (CUR != NULL))
{
If (DATA (CUR) == X)
FOUND = true;
Elseif (X < DATA (CUR))
{
PARENT = CUR;
CUR = LPTR (CUR);
D = 'L'
}
Else
{
PARENT = CUR;
CUR = RPTR (CUR);
D= 'R'
}
If (found == false)
{
Printf ('NODE NOT FOUND');
Return;
}
}
```

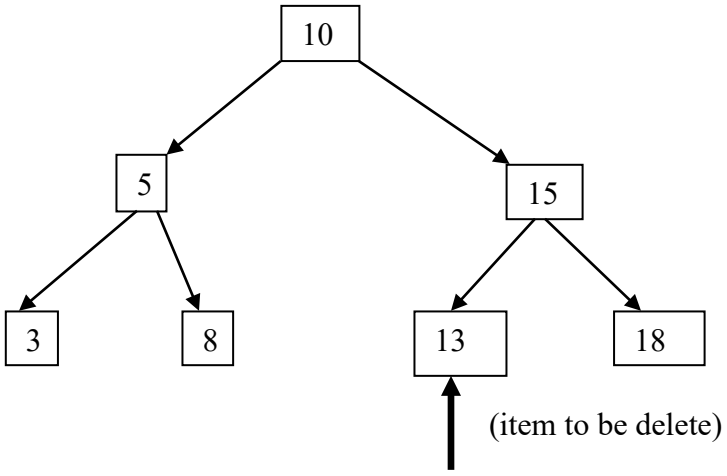
3. *[perform the indicated deletion and restructure the tree]*

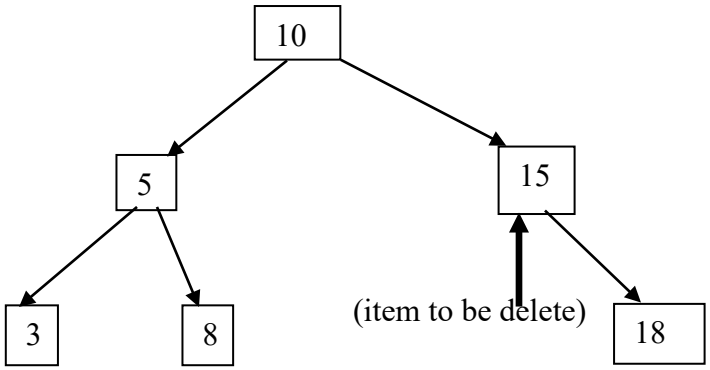
```
If (LPTR (CUR) == NULL)
Q = RPTR (CUR);   /* empty left sub tree */
Elseif  (RPTR (CUR) == NULL)
Q = RPTR (CUR);  /* empty right sub tree */
Else     /* check right child for successor */
```

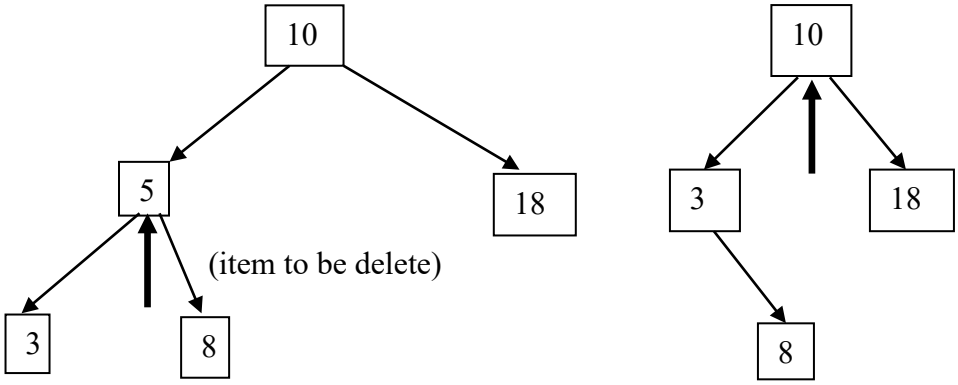
```
SUC = RPTR (CUR);
If (LPTR (SUC) == NULL)
{
LPTR (SUC) = LPTR (CUR)
Q = SUC;
}
Else     /* search for successor of CUR */
{
PRED = RPTR (CUR);
SUC = LPTR (PRED);
While (LPTR (SUC) != NULL)
PRED = SUC;
SUC = LPTR (PRED);
/* connect successor */
LPTR (PRED) = RPTR (SUC);
LPTR (SUC) = LPTR (CUR);
RPTR (SUC) = RPTR (CUR);
Q= SUC;
/* connect parent of X to its replacement */
If D = 'L'
LPTR (PARENT) = Q;
Else
RPTR (PARENT) = Q;
Return;
}
```

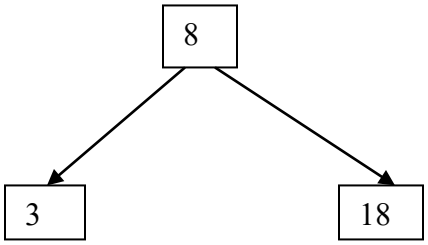
Example

In order to understand process of deletion we consider a Binary tree. Now we delete successively the nodes with keys ***13, 15, 5, and 10***. The resulting trees are as follows:









(Final tree after the deletion of all the key items)

8.8 RECONSTRUCTION OF BINARY TREE

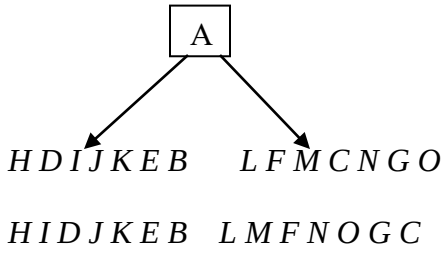
We have discussed already about the traversal of the binary tree. There were three basic methods for binary tree traversing namely *In-Order, Post-Order* and *Pre-Order*. The *In-Order, Post-Order* and *Pre-Order* traversal of binary tree may result in different sequence of nodes but by using *In-order* with preorder or *Post-order* we can uniquely find the tree. As a result original tree cannot be reconstructed, given its *In-Order, Post-Order* and *Pre-Order* traversal alone. However, if sequence of nodes produced by *In-Order* and *Post-Order* traversal of a binary tree are provided then a unique binary tree can be reconstructed. Consider the following example which illustrates the reconstruction of a binary tree given its *In-Order* and *Post-Order* traversal.

Example

The given *In-order sequence: H D I J E K B A L F M C N G O*

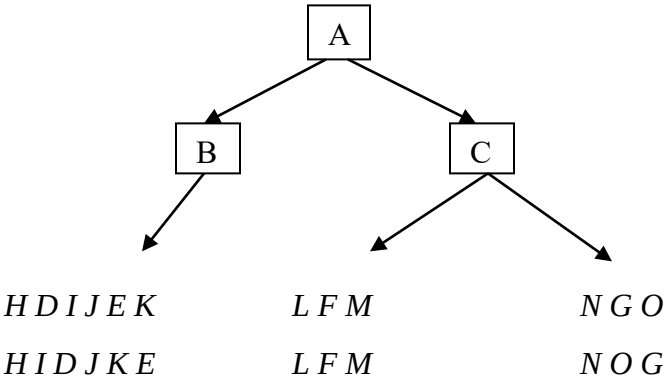
The given *Post-order sequence: H I D J K E B L M F N O G C A*

Since the first node visited in *Post-order* traversal of a binary tree is the left node, the root of the binary tree becomes *A*. In reconstruction of binary tree from *In-order* and *Post-order*, the first node is taken from the right hand side of the *Post-Order* sequence, i.e. *A* as:

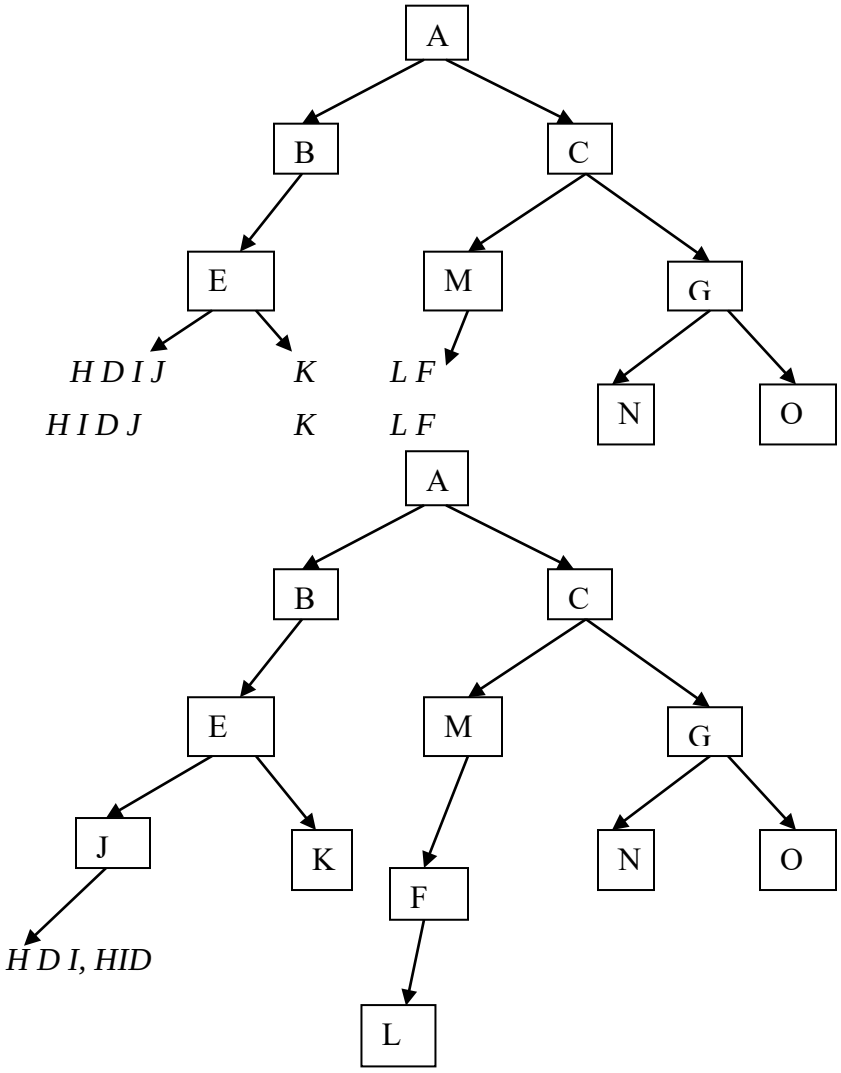


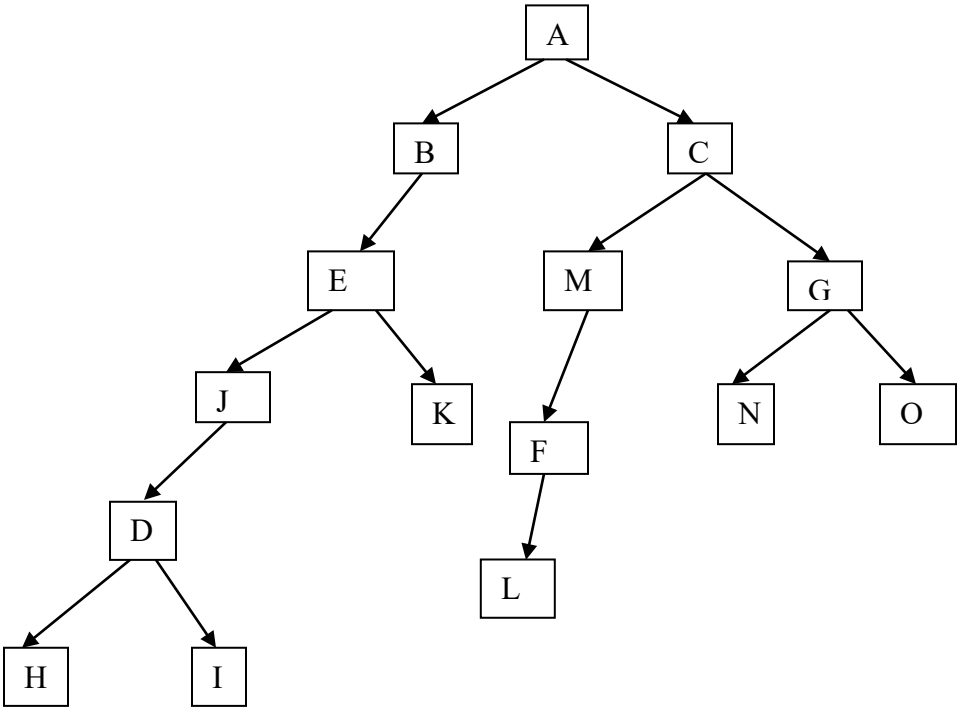
Therefore the nodes to the left of *A* in the given *In–order* sequence belong to the left sub-tree and nodes to the right of *A* belong to the right sub-tree. Moreover, the order in which the nodes to the left of *A* occur in the given *In-order* sequence is the same as the *In-Order* sequence of the left sub-tree.

Now the same scheme is applied to both the left and right sub-tree once again. Therefore the left sub-tree i.e. *In-order sequence* is *H D I J K E B* and *Post-order* sequence is *H I D J K E B*. From the *Post-order* sequence the root of this sub-tree is *B*. The *In-order* sequences of the left and right sub-tree of the sub-tree rooted at B are *H D I J K E* and *H I D I K E*. It can see as:



Hence on continuing the same set of operations in each sub-tree, the tree can be reconstructed. Further again from the *Post-Order* sequence, the root of this sub-tree is *E*. The position of *E* in *In-order* sequence determines its position on the left sub-tree rooted at *B* whereas again, looking in the *Post-Order* sequence we find *K* as the next root and its order in *In-order* sequence determines its position in the right sub-tree rooted at *E*. Similarly, the steps are repeated for all the remaining left and right sub-trees. Therefore, the reconstruction of the tree can be seen as:





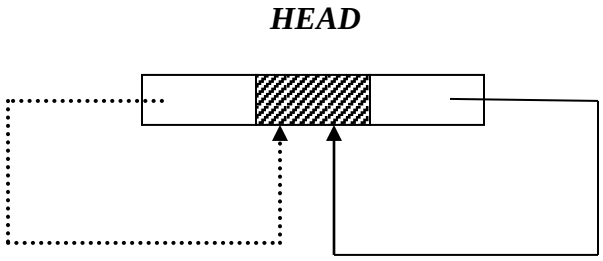
Reconstruction of the binary tree for the given sequence in *In-order* and *Post-Order*

8.9 THREADED BINARY TREE

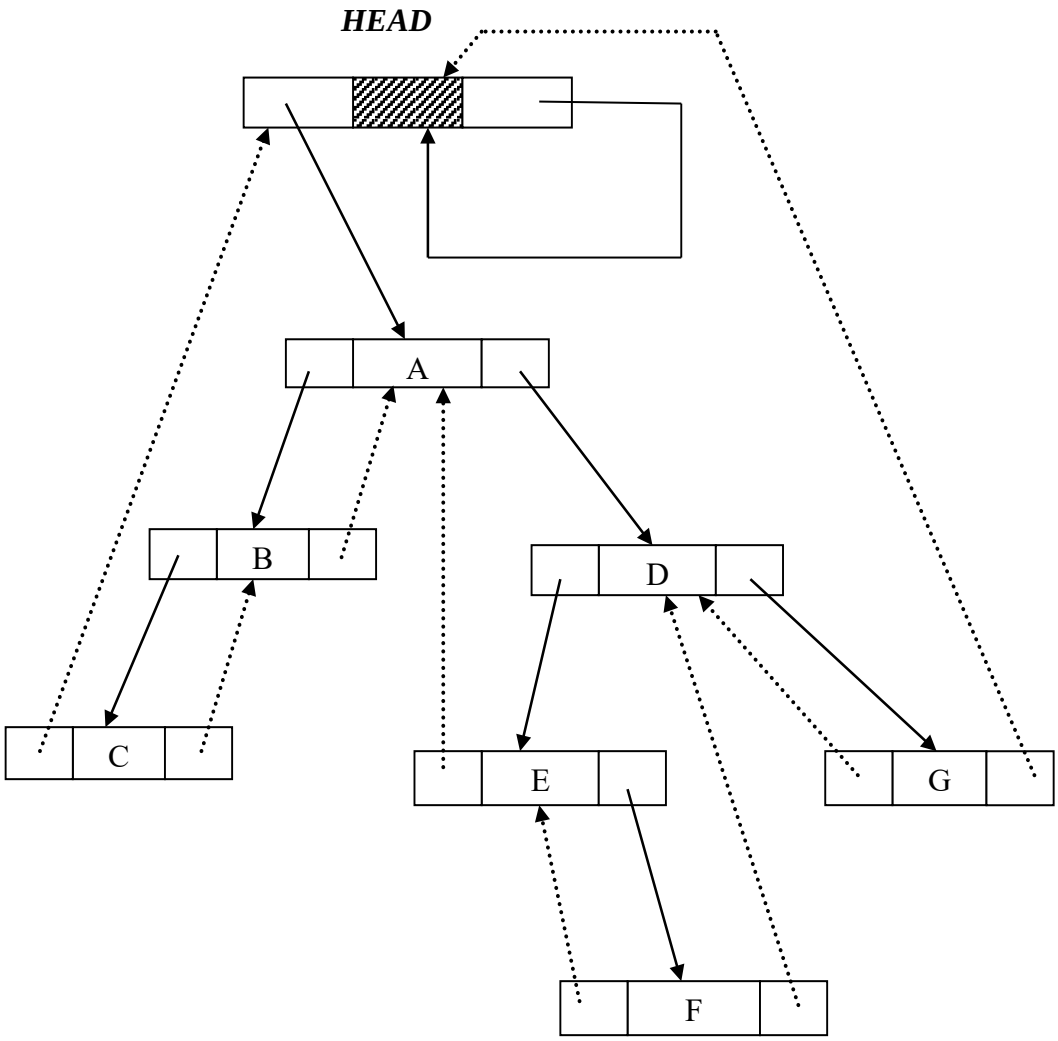
As we have seen that in a binary tree the empty sub-trees are set to *NULL* i.e. left pointer of a node whose left child is an empty sub-tree is normally set to *NULL*. Similarly, the right pointer of a node whose right child is empty sub-tree is also set to *NULL*. Thus, a large number of pointers are set to *NULL*. These null pointers can be used in different ways. Assume that the left pointer of a node ***n*** is set to *NULL* as the left child of ***n*** is an empty sub-tree, then the left pointer of ***n*** can be set to point to the ***In-order predecessor*** of ***n***. Similarly, if the right child of a node ***m*** is empty the right pointer of ***m*** can be set to point to the ***In-order successor*** of ***m***. Thus, wasted *NULL* links in the storage representation of binary trees can be replaced by ***threads.***

A binary tree is threaded according to a particular traversal order. For example, the threads for the *in-order* traversal of a tree are pointers to its higher nodes. Therefore, if the left link of a node is *NULL*, then this link is replaced by the address of the predecessor of the node. Similarly, the *NULL* right link is replaced by the address of the successor of the node. Hence the left or right link of a node can denote either a structural link or a thread; we must somehow be able to distinguish them. Here, we can make the assumption that a valid pointer values are positive and non-zero, structural links can be represented, as usual, by positive addresses. Threads on the other hand will represent by ***negative addresses***. We consider a ***Head*** node for the representation of the threaded tree. This ***Head node*** is simply another node which serves as the predecessor and

successor of the first and last tree nodes with respect to *in-order* traversal. The empty threaded binary tree can represent as:



Here the dashed arrow denotes a ***thread link***. The tree is attached to the left branch of the ***Head*** node making the pointer to the root node of the tree i.e. ***LPTR (HEAD)***. The threading of the binary tree for *in-order* traversal is given as:



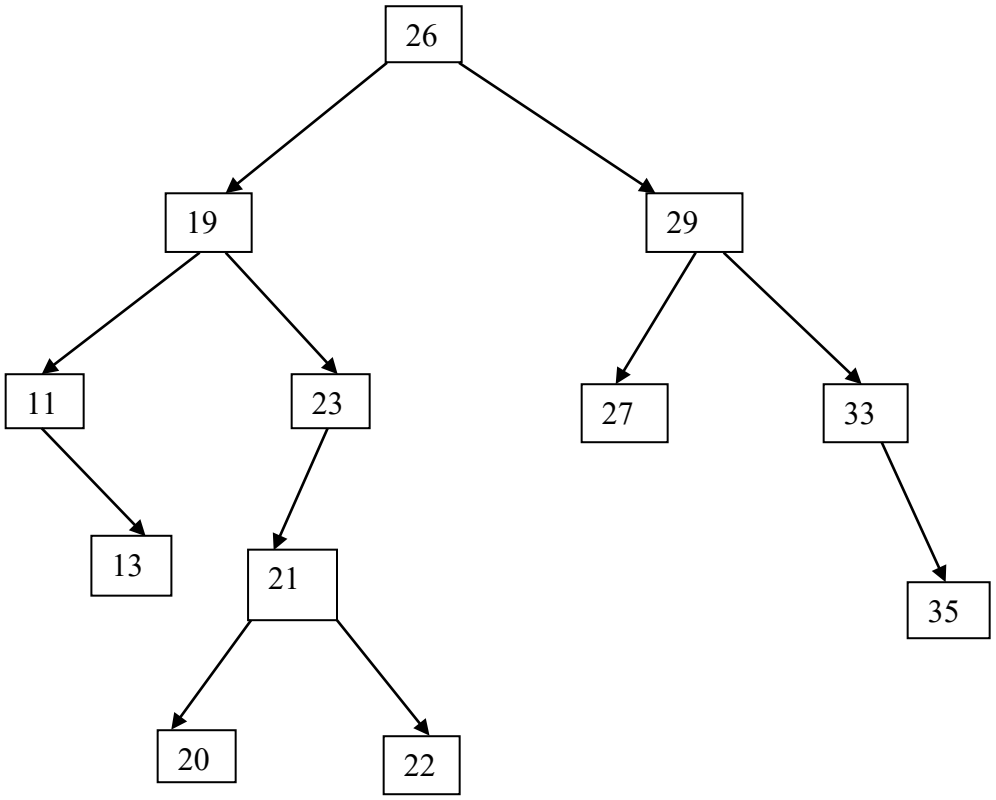
8.10 BINARY SEARCH TREE

When we place constraints on how data elements can be stored in the tree, the items must be stored in such a way that the key values in the left sub-tree of the root are less than the key value of the root, and the key

values of all the nodes in the right sub-tree of the root are greater than the key value of the root. When this relationship holds in all the nodes in the tree than the tree is called a ***binary search tree.*** Binary search trees are rooted and ordered binary trees that fulfill the following properties:

- *Key (v) ≤ Key (w) for all nodes w in the right sub tree of v,*
- *Key (v) ≤ key (w) for all nodes w in the left sub-tree if v.*

Every sub-tree of a binary search tree *T* is again a binary search tree. If *x* is a node it *T*, we denote the sub-tree of *T* with root *x* by *T (x)*. The binary search tree property enables us to output the elements sorted by key. The binary tree is travel with *In-order traversal.* The binary tree can represent as:



Binary Search Tree

In such a binary search tree if we have a given key say *K,* we can check that whether there is a node *v* in *T* with *Key (v) = k.* We just start at the root *r* of *T.* If *Key (r) = v,* then we are done. If *Key (r) < k,* then we have to look in the left sub-tree and otherwise in a right sub-tree.

8.11 OPERATION ON BST

We can perform the various operations on the binary. The first operations that we perform is the Binary search algorithm for a binary tree *T* with node *x,* and *key k.* The algorithm is defined as:

Algorithm *BST-Search* **[***This algorithm considers the two inputs. The first input is the node x and the second input is the value of the key say k. This algorithm returns a node y if Key (y) = k if such a y exists, otherwise it returns NULL.]*

```
if ((x == NULL) ||( k = key [x]))

return (x);

if (k < Key (y))

return (BST-search (LPTR (x), k));

else

return (BST-search (RPTR (x), k));
```

The second operation that we perform is to find the minimum in a binary search tree. This operation is performed by always going to the left child until we reach a node that has no left child. Now we present the algorithm to determine the minimum in a binary search tree

Algorithm *BST-minimum* [*This algorithm returns a node that has the minimum key. Here we consider the input a node x in a binary tree T. The output of the algorithm is a node in T (x) with minimum key.]*

```
If (LPTR (x) != NULL)

Return (BST-minimum (LPTR (x));

Return (x);
```

The other operation that we perform on the BST is to determine the successor of the given node *x*. This means that we are looking for the node with smallest key that is greater than *K (x)*. This successor is completely determined by the position in the tree, so we can find it without comparing any elements. If a node *x* has a right child, that is *RPTR (v) ≠NULL,* then the successor simply is the node in *T (RPTR (x))* with minimum key. If *x* does not have a right child, then the successor of *x* is the lowest ancestor of *x* that has a left child that is also an ancestor of *x*. BST-successor computes the successor of a node *x*. The second part deals with the case that *x* has no right child. We go up the paths from *x* to the root and *y* is always the parent of *x*. We stop when either *y = NULL* or *x = LPTR (y)*. Now we present the algorithm for determining the successor of a given node from the binary search tree.

Algorithm *BST-successor* *[This algorithm returns the successor of a given node x in a binary search tree T. The input to this algorithm is the node x for the given binary search tree. The output of this algorithm is the successor of node x, if x has it, otherwise it returns the value NULL.]*

```
If (RPTR (x) != NULL)

Return (BST-minimum (RPTR (x));

y= parent (x);
```

While ((y != NULL) && (x != LPTR (y))

x= y;

Y = parent (y)

Return (y);

8.12 AVL TREE

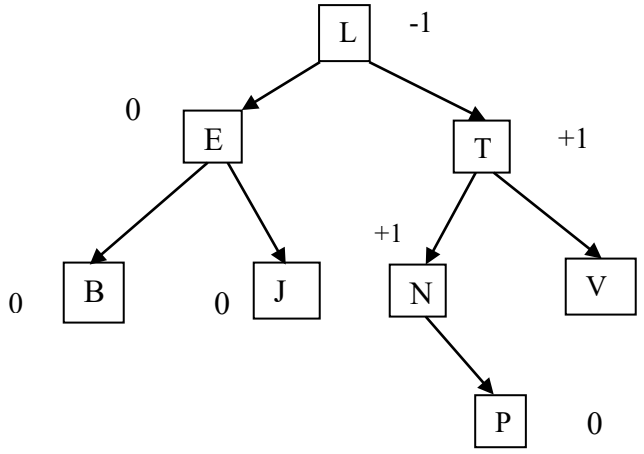
An **AVL** tree is binary search tree with the additional balance property that, for any node in the tree, the height of the left and right sub-tree can differ by at most 1. As usual, the height of an empty sub-tree is -1. Therefore, the first balanced binary search tree was the **AVL** tree (named after its discoverers, *Adelson-Velskii and Landia*), which illustrate the ideas that are thematic for a wide class of balanced binary search tree. Thus, it is binary search tree that has an additional balanced binary condition. The simplest idea is to require that the left and right sub-tree have the same height.

In other words we can say that an **AVL tree** is a binary search tree with the extra property that for every internal node *x,* the height of the sub-tree with root *LPTR (x)* and the height of the sub-tree with root *RPTR (x)* differ by at most one. The **AVL tree** has the following properties:

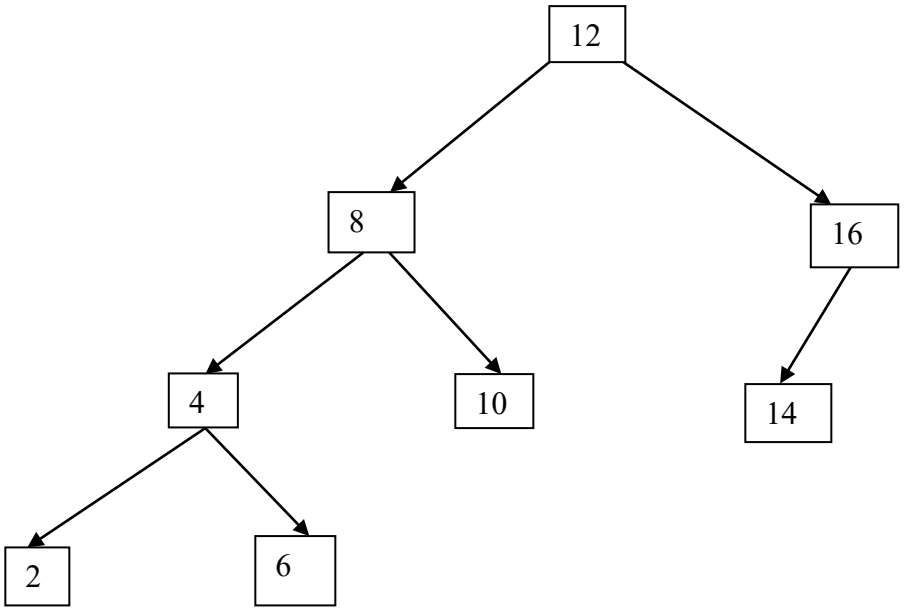
1. An **AVL tree** fulfills the binary search tree property.
2. For every internal node say *v, Bal (v) €{-1, 0, 1}.*

Now we define more precisely about the notation of a "*balanced tree*". The height of a binary tree is the maximum level of its leaves (***depth of tree***). The height of *null* tree is defined as ***-1***. Thus, a balanced binary tree or **AVL tree** is a binary tree in which the heights of two sub-trees of every node never differ by more than ***1.***

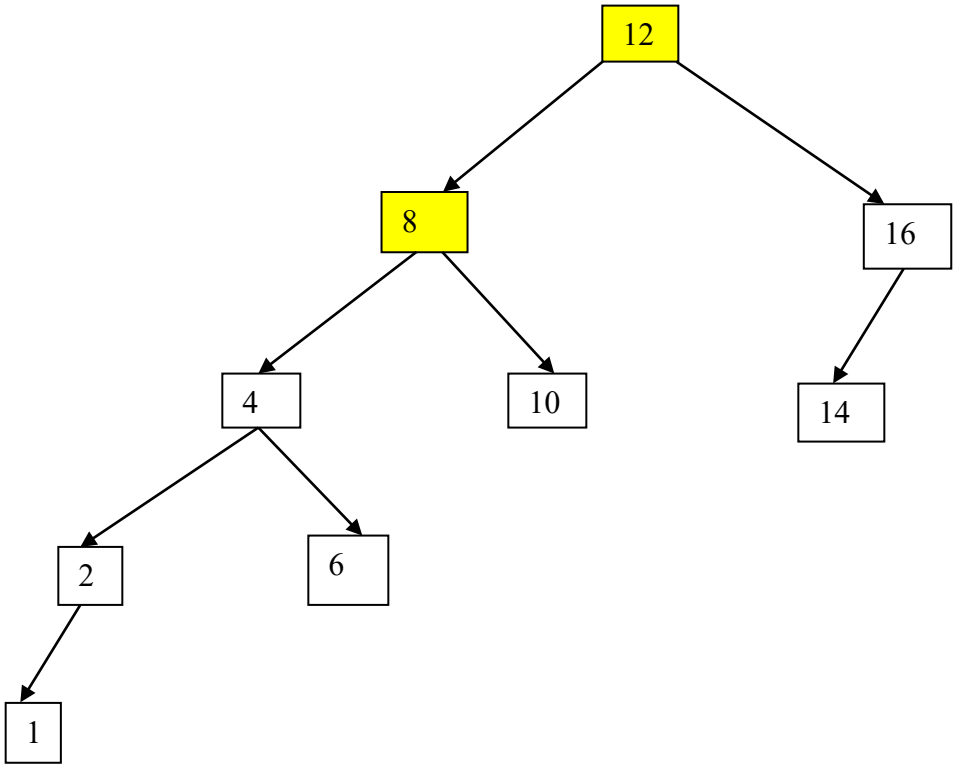
The ***balance factor*** of a node in a binary tree can have value 1, -1 or 0 depending on whether the height of its left sub-tree is greater than, less than or equal to the height of its right sub-tree. The balance factor of each node is defined as follows:



We c onsider t he a nother e xample t o hi ghlight the di ffence bw teen a n ordinary b inary s earch t ree an d t he **AVL tree.** Let us c onsider t he following two binary search trees.



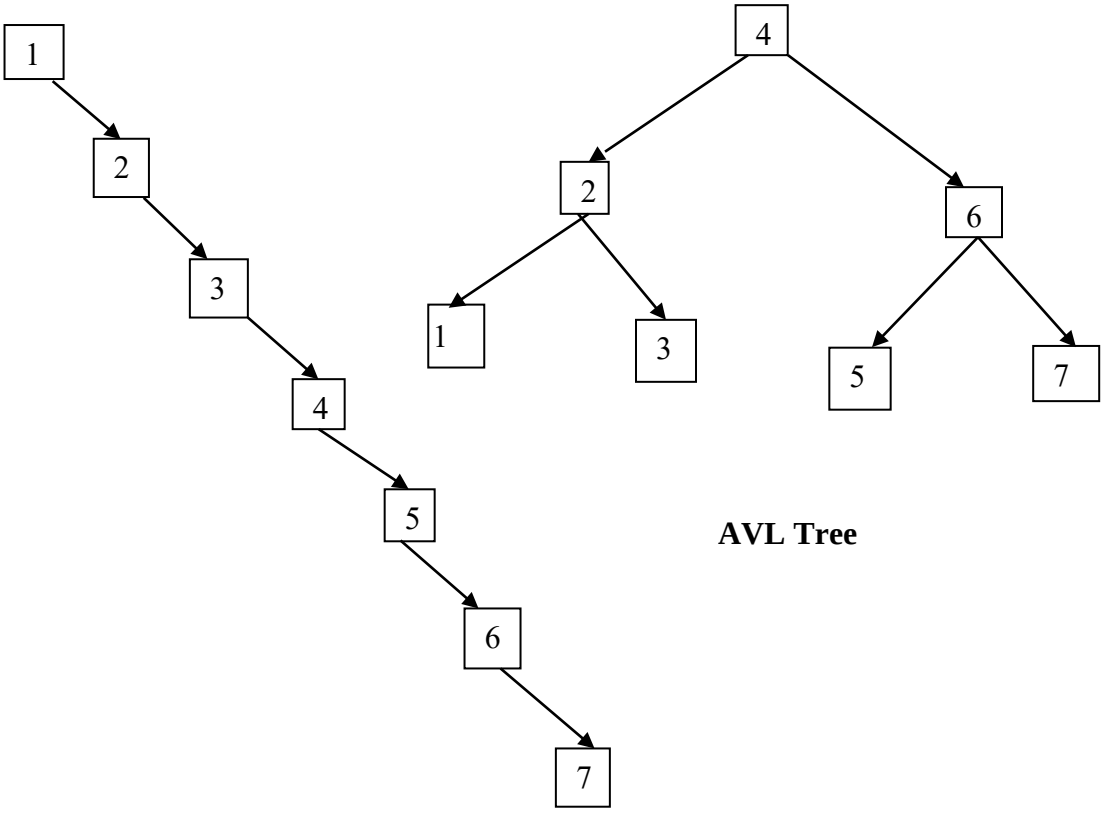
Binary Search Tree (A)



Binary Search Tree (B)

The b inary t ree **(A)** satifies t he **AVL** balance condition a nd i s t hus a n **AVL Tree.** The b inary search t ree (**B)** is not t he **AVL tree** because t he shaded nodes have left sub-tree whose heights are 2 larger than their right sub-trees. If a new node say 13 is inserted then node 16 would also violate the conditon for b alancing b ecause t he l eft sub-tree w ould h ave h eight 1, while the right sub tree would have height -1.

The m ajor advantage of a n **AVL tree** is t hat t here are h eight b alanced tresses so that the operations like insertion and deletion have O(log n) time complexity. Let us consider an example of the binary tree with keys: 1, 2, 3, 4, 5, 6, 7, t he binary tree and corresponding **AVL Tree** for these keys can represent as:



AVL Tree

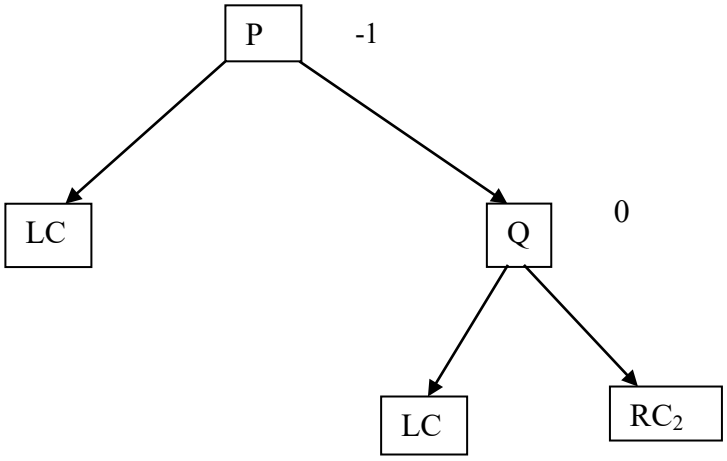
Binary Tree

8.13 OPERATION IN AVL TREE

Here we are describing the two basic operations on the **AVL Tree,** namely, the operation of insertion and deletion of the node in or from the **AVL Tree.**

Insertion

To di scuss t he i nsertion ope ration f or t he **AVL Tree** we co nsider an example. Now consider the binary search tree with the balance factor as:



Binary Search Tree with Balance factor

Here, the balance factor of **P** is **-1** and that of **Q** is **0**. $\mathbf{LC_1}$ is the left child of **P** and $\mathbf{LC_2}$ and $\mathbf{RC_2}$ are the left and right children of the node **Q**. After insertion there are two cases that can make the tree unbalance. These cases are as follows:

(a) The new node is inserted as a child (left to right) of the leaf node of sub-tree $\mathbf{RC_2}$ as shown in figure (a)

(b) The new node is inserted as a child (left to right) of the leaf node of sub tree of structure $\mathbf{LC_2}$ as shown in figure (b)

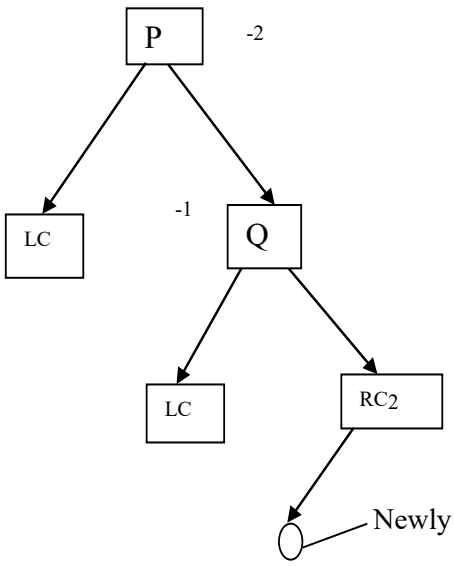


Figure (a)

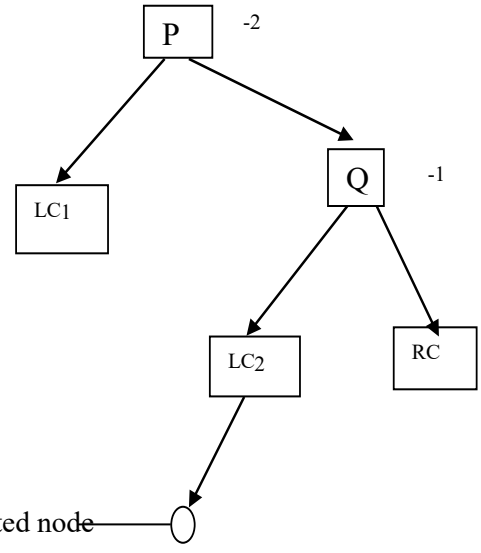
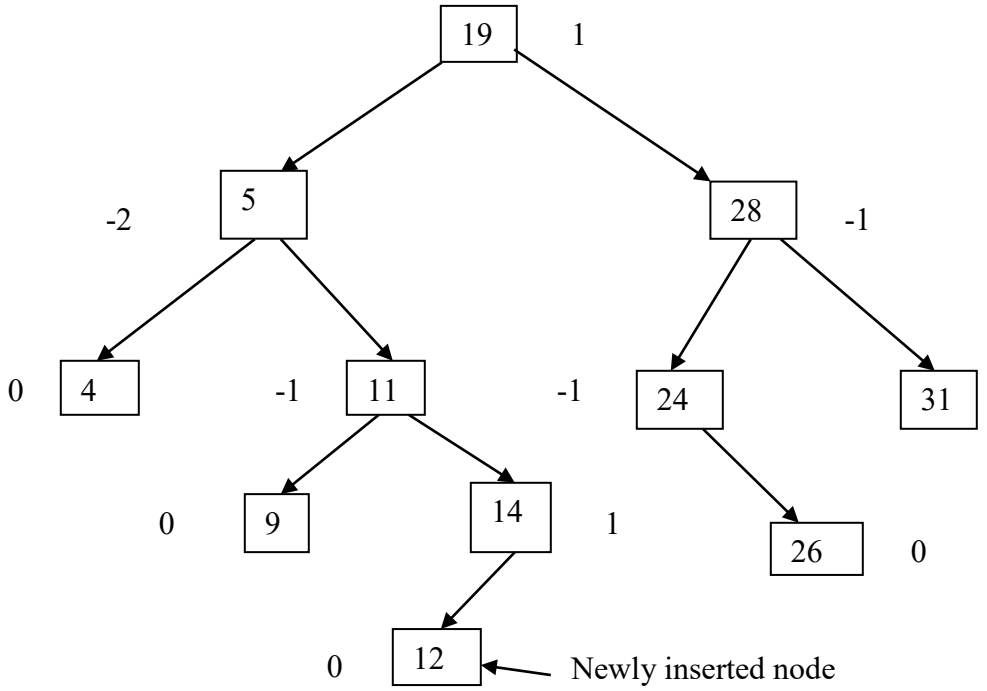


Figure (b)

Now to accomplish the balance in both these cases we consider the following:

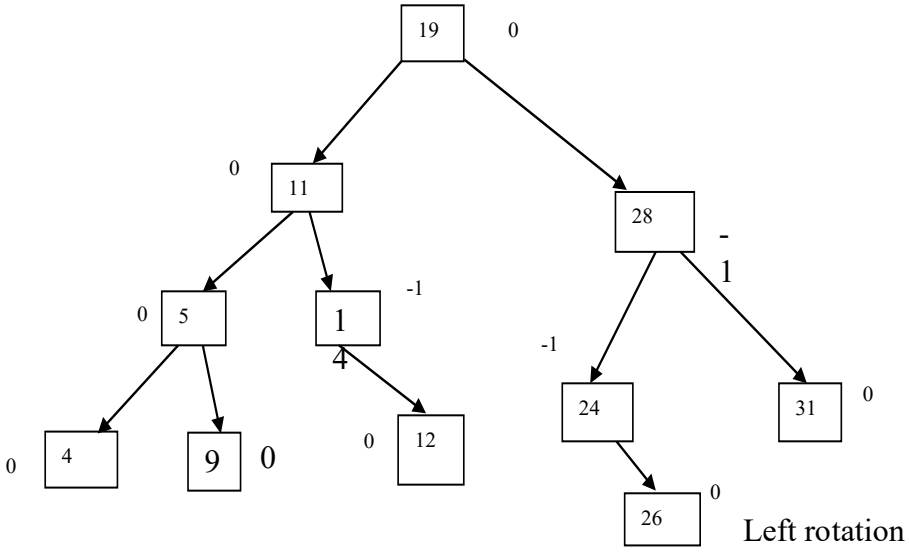
RIL-102

Case (a) Consider the given tree:



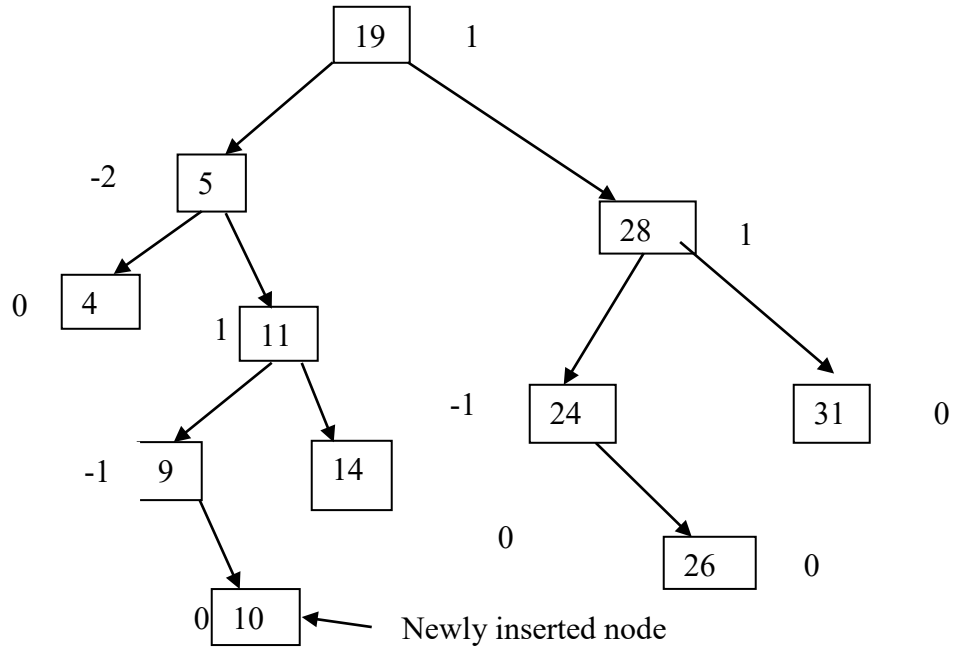
Insertion of a new Node

As we can see from this binary search tree that on insertion of the new node, the balance factor of the node containing the data **5** violates the condition of an **AVL tree**. Now to rebalance the tree, we require making a **left-rotation** of the tree along the node containing the data **5** as the left child containing the data **11** and the node containing the data **9** as the right child of the node containing the data **5**. This can show as follow to consider the case (a).



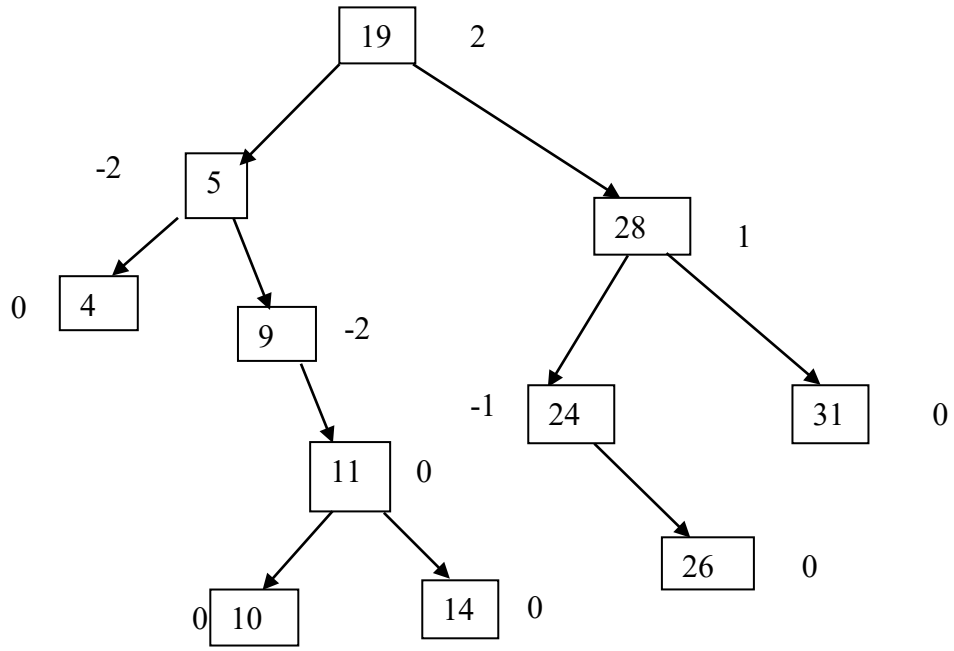
Rebalancing the Tree using Left Rotation

Case (b): Now suppose instead of **12**, we insert a node with value **10.** This node would get inserted as the right child of the node containing value **9**. After this the tree no longer remains a balanced tree as the balance factor of node containing value **5** breaks the rules of **AVL tree** as show follows:



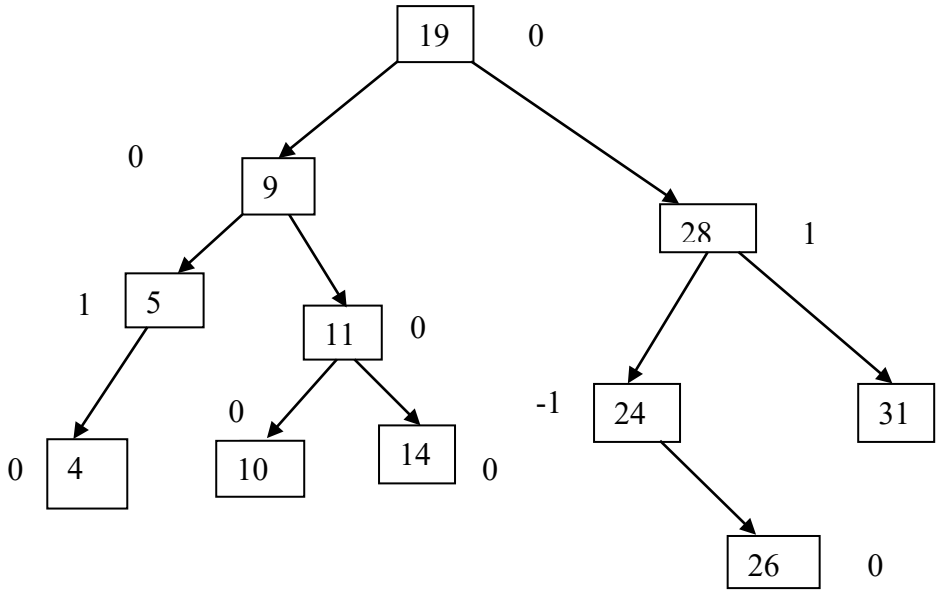
Unbalance tree after addition of New Node

Now the tree will right roatate for its rebalancing.



Rebalancing the tree again using Right Rotation

We can see that even after right rotation, the tree remians unbalanced and hence, it is rotated to left along the node **5**. As a result node **9** becomes the left child of node **19**. Node **5** becomes the left child of node **9.** Since there is no left child for node **9** the right child of node **5** is empty. Thus finally, tree becomes a balanced binary tree or an **AVL Tree.** This procedure of rotating the tree, first to the right and then to the left is known as **double rotation**. The resultant tree can show as:



Balance tree after double rotation

Delete Operation

Deletion is performed similar to insertion. We use the delete method from binary search tree and then move the path up to the root and try to restore the **AVL Tree** property.

When we delete a node from a binary search tree the following three cases can occur:

- It is a leaf node of the binary search tree. Thus, it is an internal node with two virtual leaves in our **AVL Tree** with balancing factor 0.
- It is an internal node with only one child. Thus, in an **AVL Tree,** this means that one of its children is a virtual leaf.
- It is a node note with two children that are internal nodes, too. Then we delete its successor and copy the content of the successor to the node.

8.14 B TREE

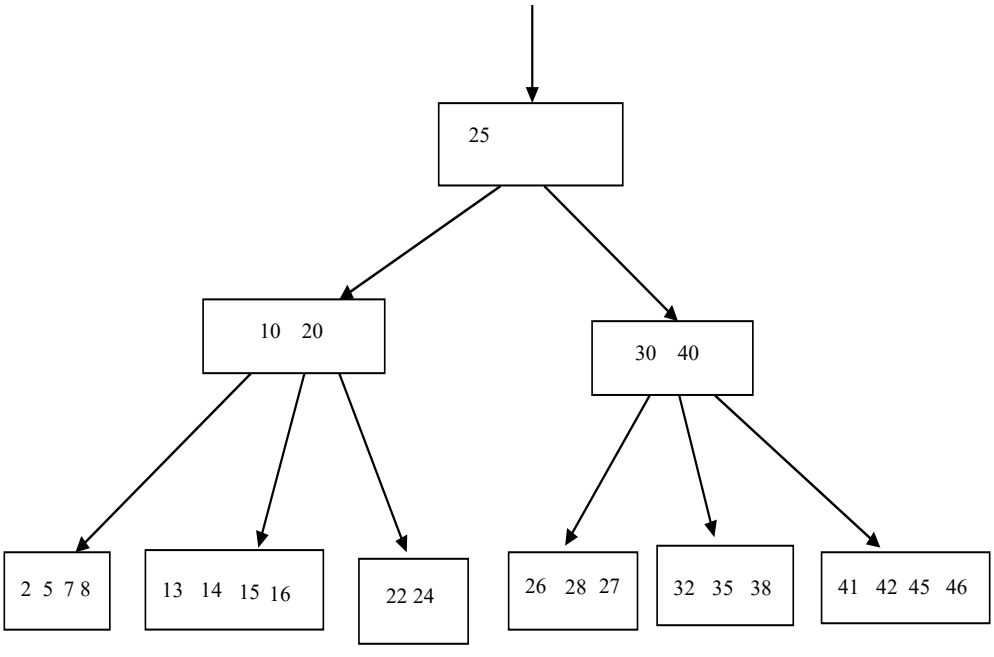
To minimize the search in the binary tree our requirement is to make the height of the tree as small as possible. We can accomplished this by ensuring first that no empty sub-trees appear above the leaves, all leaves be on the same level and every node expect the leaves has at least some minimal number of children. Hence we require that each node has at least as many children as the maximum possible. These conditions lead to the following definition:

A ***B-tree*** of order **m** is an **m-way** tree in which:

- All leaves are on same level.
- All internal nodes except the root have at most **m** (non-empty) children, and at least [**m/2]** non-empty children.
- The number of keys in each internal node is one less than the number of its children, and these keys partition the keys in the children in the fashion of a search tree.
- The root has at most **m** children, but may have as few as 2 if it is not a leaf, or none if the tree consists of the root alone.

Example

Let us consider a **B-Tree** of order 5 with 3 levels. All pages contain 2, 3, or 4 items. The exception is the root which is allowed to contain a single item only. All leaf pages appear at level 3. This **B-Tree** can represent as:



RIL-102

B-Tree of order 5 with level 3

8.15 INSERTION IN A B-TREE

The **B-Tree** are not allowed to grow at their leaves instead they are forced to grow at the root. The general method of insertion is as follows:

- First a search is made to see if the new key is the tree. This search will terminate in failure at a leaf.
- The new key is added to the leaf node. If the node was not previously full, then insertion is finished.
- When a key is added to a full node, then the nodes split into two nodes on the same level except that the median key is not put into either of the new nodes but instead sent up the tree to be inserted into the parent node.
- When a search is later made through a tree, a comparison with the median key will serve to direct the search into proper sub-tree.
- When a key is added to full root, then the root splits into two and the median key sent upward becomes a new root.

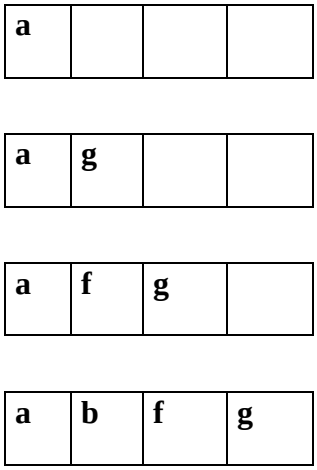
Example

Now we consider an example to understand the process of insertion in the **B-Tree.** The following keys are inserted into the **B-Tree** of order 5.

a g f b k d m j e s i r x c l n t u p

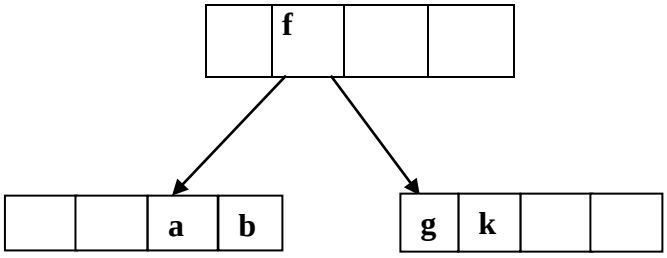
The first four keys will insert into one node, as follows:

1. **a g f b:**



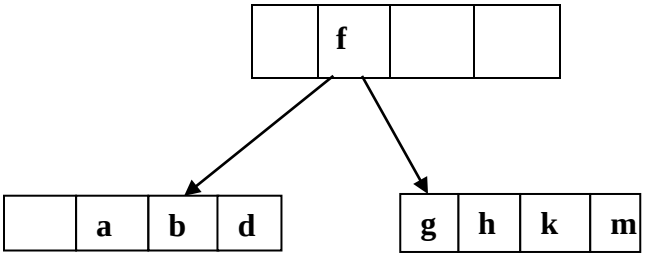
The keys are stored into proper order as they inserted.

2. **k :**



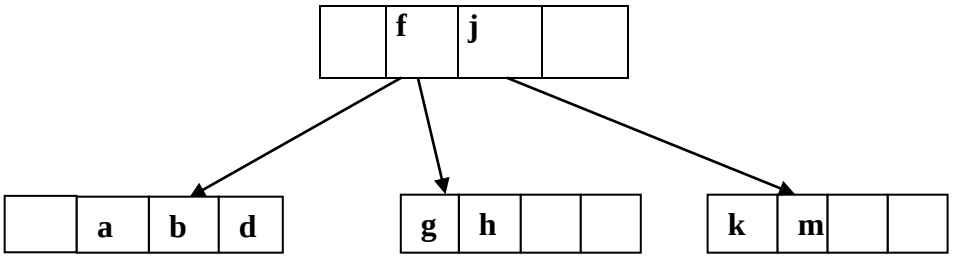
Now the insertion of key **k** causes the node to split into two and the median key **f** moves up to enter a new node.

3. **d, h, m:**



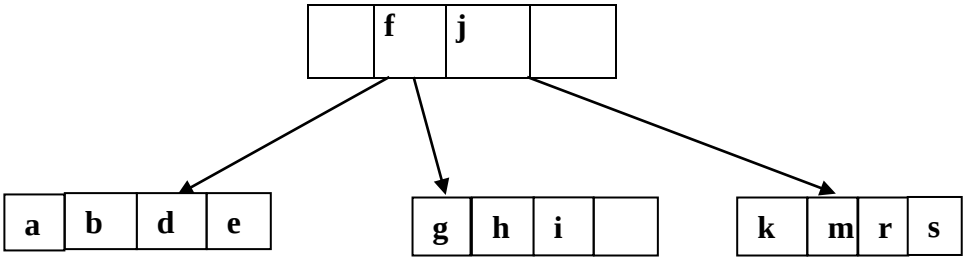
Since the split nodes are now only half full, the next three keys can be inserted without difficulty. However, these simple insertions can require rearrangement of keys within a node.

4. **j:**

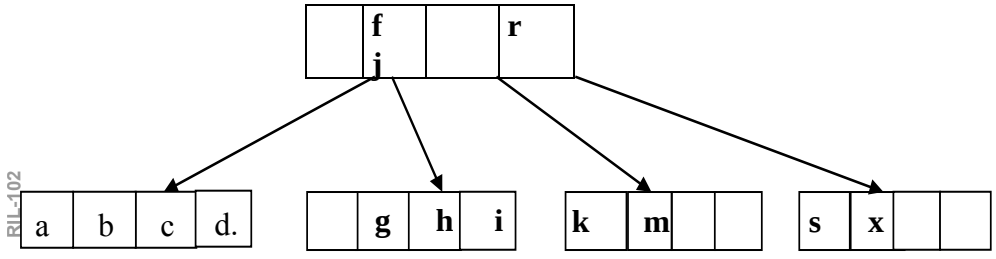


The next insertion **j** again splits a node and this time it is **j** itself that is the median key and therefore moves up to join f in the root.

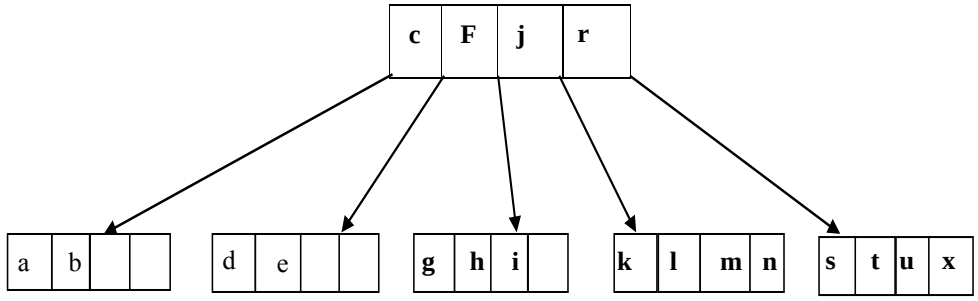
5. **e s i r :**



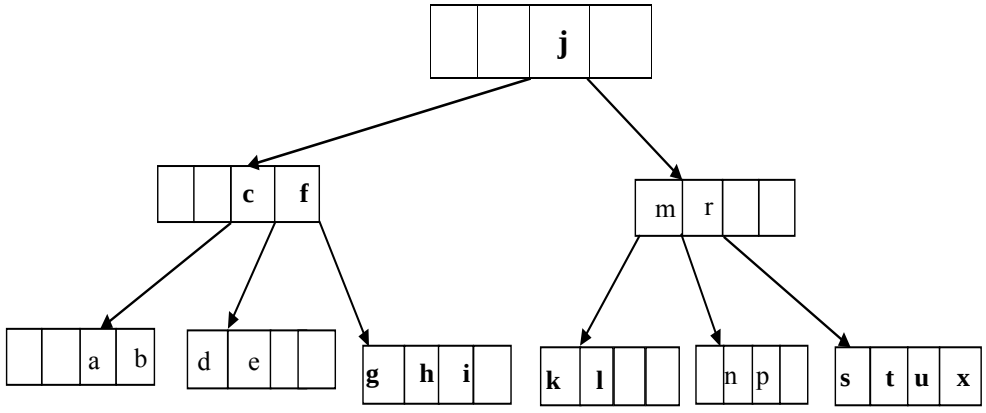
6. **x :**



7 **c l n t u:**
.



8 **p**
. **:**



The insertion **p** splits the node originally containing **k, l, m, n** sending median key **m** upward into the node containing **e, f, j, r** which is however already full. Hence, this node in turn splits and a new node containing **j** is created.

8.16 DELETE IN A B-TREE

Deletion of an item or items from a B-tree is fairly straightforward. As we have seen from the insertion operation that a new key always goes first into leaf. If the key that is to be deleting is not the leaf then its immediate **predecessor** or **successor** is promoted into the position of the deleted key. The natural order of keys is guaranteed to be in the leaf nodes. Hence, we can promote the immediate predecessor or successor into the position occupied by the deleted key and delete the key from leaf. If the leaf contains more than minimum number of keys, then one can be deleted with no further action. If the leaf contains the minimum number, then we first look at the two leaves that are immediately adjacent and children of same node. If one of these has more than the minimum number

of keys, then one of them can be moved into the parent node and the key from the parent is moved into the leaf where the deletion is occurring. If finally the adjacent leaf has only the minimum number of keys, then the two leaves and the median key from the parent can all be combined as one new leaf which will contain no more than the maximum number of keys allowed. If this step leaves parent node with too few keys, then the process propagates upwards. In this case, the last key is removed from the root and then the height of the tree decreases. Thus we may distinguish two different circumstances in the deletion process:

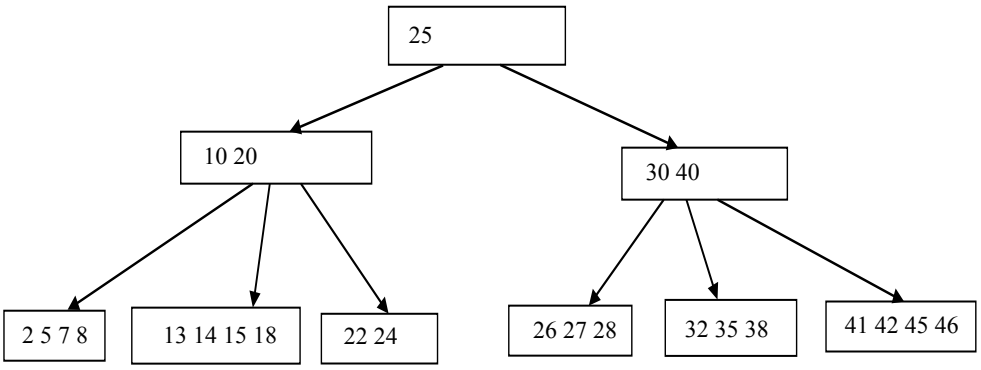
1. The item to be deleted is on a leaf page. In this case the removal algorithm is plain and simple
2. The item is not on a leaf page so it must be replaced by one of the two lexicographically adjacent items which happen to be on leaf pages and can easily be deleted.

Example

Here we consider an example for demonstrating the deletion the items or keys from the given **B-Tree.** Here we have the following keys for them the sequential deletion will occur.

Keys: 25 45 24; 38 32; 8 27 46 13 42; 5 22 18 26; 7 35 15;

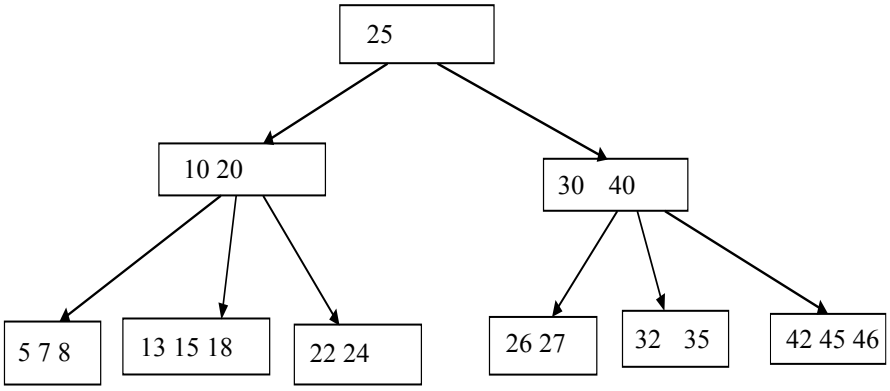
The sequential deletion of these key from the existing **B-Tree** can represent as:



Given exisitng B-Tree

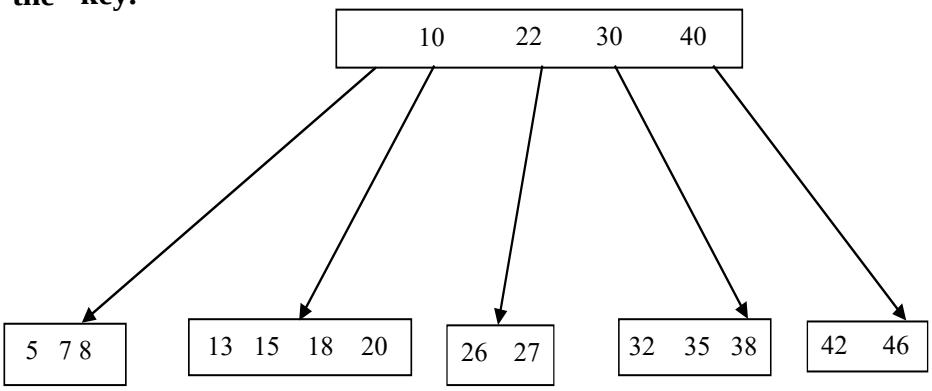
Solution Steps :

(1) Check the key from leaf, if the key that is to be deleting is not the leaf then its immediate **predecessor** or **successor** is promoted into the position of the deleted key.



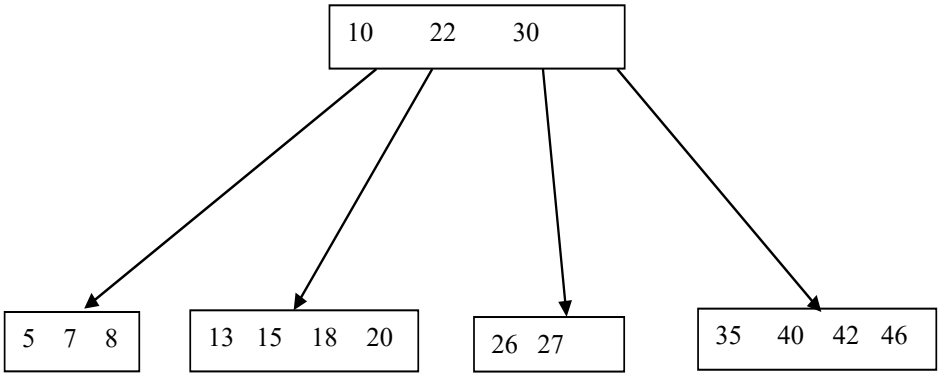
Step 1

(2) Promote the immediate predecessor opr successor into position occupied by the deleted key and delete the the key.



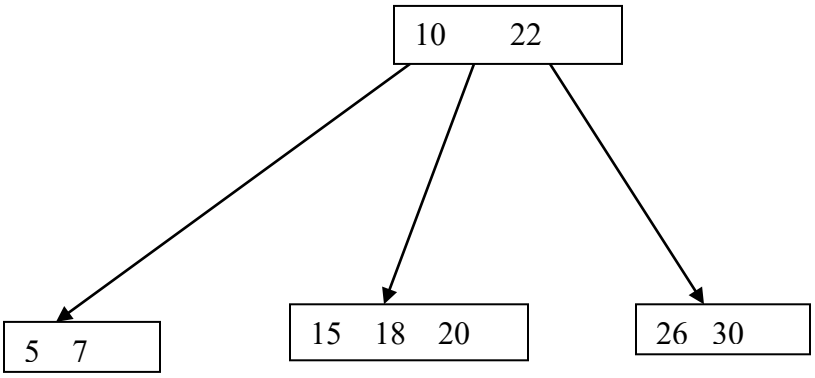
Step 2

(3) If the leaf contains the minimum number, then we first look at the two leaves that are immediately adjacent and children of same node. If one of these has more than the minimum number of keys, then one of them can be moved into the parent node and the key from the parent is moved into the leaf where the deletion is occurring.



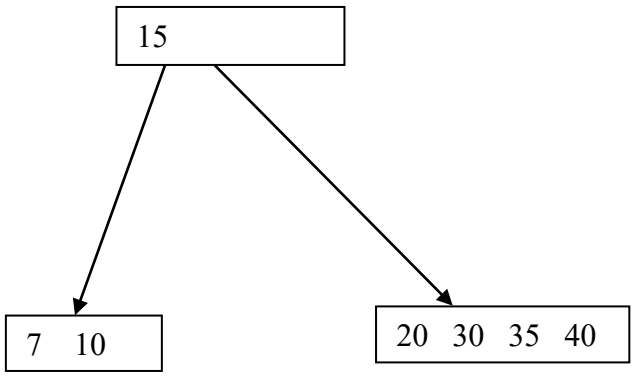
Step 3

(4) The two leaves and the median key from the parent can all be combined as one new leaf which will contain no more than the maximum number of keys allowed

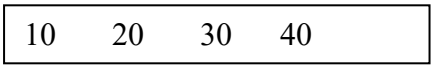


Step 4

(5) , the last key is removed from the root and then the height of the tree decreases



Step 5



Step 6

Deletion of given keys in step wise manner from the Given B-Tree

8.17 SUMMARY

In this unit we described a very important non-linear data structure i.e. T ree. T he t ree s pecifies t he d ynamic m emory allocation. A Tree consists of a set of nodes and a set of directed edges that connect pairs of nodes. Trees are u seful i n d escribing an y structure w hich i nvolves hierarchy. Various operations like insertion, deletion, copy and traversal of the t ree w ere di scussed i n f ull de tail. E nough examples w ere us ed t o

describe the tree and its various operations. The contents of this unit can be summarized as:

- Tree is the nonlinear data structure.
- A tree consists of a root node and a number of sub-trees.
- There are different types of trees showing different types of properties.
- There are many terms associated with a binary tree like its right and left child, sub-tree, in-degree, out-degree, successor, predecessor and leaf nodes.
- The binary tree can represent with sequential data structure like array and with non-contiguous data structure like linked list.
- There are various operations can performs on the Binary tree like insertion, deletion, searching and traversing.
- A fundamental and important operation on the tree is traversal. Traversal of a binary tree implies visiting every node of that tree exactly once in some specified sequence. There are three standard traversal techniques for binary tree. These techniques are: In-order, Post-Order and Pre-Order.
- The binary tree can construct from the given sequence of traversing in the form of In-Order and Post-Order.
- The recursive and iterative methods can use for the traversing in any specified order.
- In a binary tree, many nodes have one child or no children. The pointers for the empty children for these nodes are set to *NULL*. A more effective utilization of these pointers is possible if *NULL* left pointers are set to point to the In-Order predecessor of that node and *NULL* right pointers are set to point to the In-Order successor of that node. These pointers, so introduced are called *threads*. Threads help in writing non-recursive version of In-Order, Post-order and Pre-Order traversal.
- When we place constraints on how data elements can be stored in the tree, the items must be stored in such a way that the key values in the left sub-tree of the root are less than the key value of the root, and the key values of all the nodes in the right sub-tree of the root are greater than the key value of the root. When this relationship holds in all the nodes in the tree than the tree is called a ***binary search tree.***
- Operation of insertion and deletion can describe for the binary search tree.

- **AVL Trees** are height balanced trees where the difference between heights of left and right sub-trees rooted at any node cannot be larger than one.
- The fundamental operations, namely searching, insertion and deletion can be performed more efficiently on **AVL Trees.**
- In an **AVL Trees** the only case that causes difficulty when the new node is added to sub-tree of the root that is strictly taller than other sub-tree then the height is increased. This would cause one sub tree to have height 2 more than other; whereas the **AVL Trees** condition is that the height difference is never more than1.
- When an **AVL Trees** is right high the action needed in this case is called left rotation. On the other hand when the tree is left high, the right rotation is performed. In some cases, the tree is needed to rotate twice, and then the condition is called double rotation.
- A **B-Tree** of order *m* is an *m-way* search tree where each node except the root must have *m / 2* children at most "*m*" children. The root node is allowed to have two children at the minimum.
- The searching insertion and deletion operation on a **B-Tree** are same as that of 2-3 trees.
- It is interesting to note that **B-Tree** grows or shrinks upwards during insertion and deletion of key values.

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SELF-EVALUATION

Multiple Choice Questions:

1. The in-order traversal of some binary tree produced sequence **DBEAFC**, and the post-order traversal of the same tree produced the sequence **DEBFCA,** which of the following is a correct pre-order traversal sequence:

   a. **DBAECF**

   b. **ABEDFC**

   c. **ABDECF**

   d. None of the above

2. A balanced binary tree is a binary tree in which the heights of two sub-trees of every node never differ by more than:

   a. 2

   b. 1

   c. 3

   d. None of the above

3. Which of the following statement is TRUE in view of a complete binary tree?

   a. The number of nodes at each level is 1 less than some power of 2.

   b. The out degree of every node is exactly equal to 2 or 0

   c. Total number of nodes in the tree is always some power of 2

   d. None of these

4. Level of any node of tree is:

   a. The distance from the root.

   b. Height of its left sub-tree minus height of its right sub-tree.

   c. Height of its Right sub-tree minus height of its left sub-tree.

   d. None of the above

5. What is the minimum of nodes required to arrive at a **B-tree** of order 3 and depth 2? Assume that the root is at depth 0.

   a. 4

   b. 5.

   c. 12.

d. 13

6. The minimum height of a binary tree having 1000 nodes would be

   a. 10

   b. 11

   c. 100

   d. None of the above

7. The number of trees possible with 10 nodes is:

   a. 1000

   b. 1200

   c. 1014

   d. None of the above

8. If in a given directed tree, the out-degree of every nodes is less than or equal to *m*, then it is called a:

   a. Threaded binary tree

   b. Complete *m-ary* tree

   c. *m-ary* tree

   d. *None of these*

Fill in the blanks

1. In a B-tree of order n, each non root node contains at least …… keys. (n / n/2)

2. The minimum number of keys contained in each non-root node of a B-tree of order 15 is…… (7 / 9)

3. A B-tree of order n is also called…… (n-(n-1) / n + (n-1)

4. A……is a complete binary tree where value at each node is at least as much as values at children node. (heap / B-tree)

5. In a B-Tree of order m, no node has more than ……. Sub tree. (m / n)

6. The maximum level of any leaf in the tree is also known as----- of the tree.

7. In an AVL tree the heights of the two sub trees of every node never differ by more than….

8. In a tree, a node that has no children is called ….. node

9. A binary tree is threaded for a post order traversal, a NULL right link of any node is replaces by the address as its…….

10. The number of nodes which must be traversed from the root to reach a leaf of a tree is called the ….. of a tree.

State whether True or False

1. B-trees are balanced trees.

2. Total number of cycles contained in any tree is 0

3. The process of determining predecessor and successor nodes, for any given node is easy for unthreaded tree compared to threaded counterparts.

4. The in-degree of the root node in any binary tree is 1.

5. If Pre-order and post-order of any binary tree re known than the tree can easily draw.

6. AVL – tree is not a complete binary tree.

7. Binary search tree is a special case of the B-tree

Answer the following questions

1. Define degree of B-tree.

2. What is complete binary tree? What is in degree and out degree of a complete binary tree?

3. What is AVL Tree? Show the insertion and deletion operations in AVL tree with the help of suitable example.

4. Write a ‘ C’ function to formulate the iterative algorithm for traversing a binary tree in in-order and post-order

5. How many different directed trees are there with three nodes?

6. Give a directed tree representation of the following formula: (a +b) * (c + d) ↑ e

7. Show that in a complete binary tree, the total number of edges is given by 2 (n_t-1), where n_t is the number of terminal nodes.

8. Write a ‘ C’ program to construct a lexically ordered binary tree. Also show the operation of insertion and deletion of a node from it.

9. Write a function in ‘C’ for finding the k^{th} element in the pre-order traversal of a Binary tree.

10. Write a function in ‘ C’ language to search an element from the binary search tree.

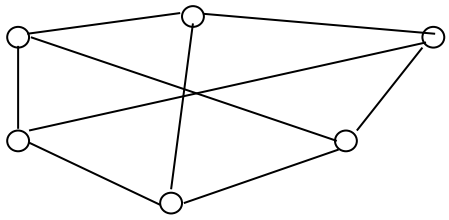
11. Compare and describe the advantages and disadvantages of AVL tree and B-tree.

UNIT-9 GRAPH

Structure

9.0 INTRODUCTION

This unit consists with the concept of graphs in general as well as in mathematical form and its basic terminology. It includes the method for the graph representation like matrix representation or adjacency matrix form with linked list. It covers the various algorithms for searching the graph like breadth first search and depth first search. The concept of spanning tree for obtaining the optimal path from the graph is also introduced and implemented with Kruskal's and prism's algorithms. The technique for obtaining the shortest path from the graph is implemented with Bellman ford algorithm, Dijkstr's algorithm and Floyd-Warshall algorithm. A Graph is a nonlinear data structure that is used to represent a relational data e.g. a set of terminals in a network or a road map of all cities in a country. Such type of complex relationship can only be represented using a graphs data structure. Thus, a graph is such type of a non linear data structure which is having point to point relationship among the nodes. A tree can view as a restricted graph. Each node of the graph is called as a *vertex* and link or line drawn between them is called *edge*. A general graph can view with the following diagram which consists with 6 (six) nodes or vertices and eight (08) links or edges.



9.1 OBJECTIVES

After reading this unit the learner is able to do the following task.

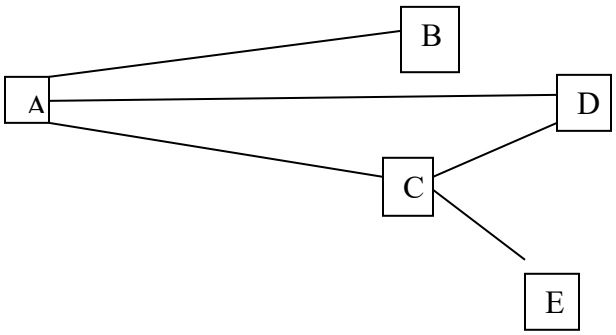
- Interpret the graph mathematically.
- Represent the graph as adjacency matrix and path matrix
- Perform the search in the graph either in breadth first or depth first search manner
- Able to construct spanning tree from the given graph using Kruskal's and Prim's algorithms
- Able to explore the shortest path from the given graph

9.2 MATHEMATICAL DEFINITION OF THE GRAPH AND ITS BASIC TERMINOLOGY

Mathematically, A graph ***'G'*** consists of two sets ***V*** and ***'E'*** such that ***G={V, E}***,Where ***V*** is finite nonempty set of vertices or nodes, ***V(G)*** represents set of vertices, ***E*** is a set of edges and ***E(G)*** represents set of Edges. Therefore we can say that a graph ***G*** consists of a non empty set ***V*** called the set of nodes (points, vertices) of the graph, a set ***E*** which is the set of edges of the graph, and a mapping from the set of edges ***E*** to a set of pairs of elements of ***V***.

Example

Let us consider the graph as follows:



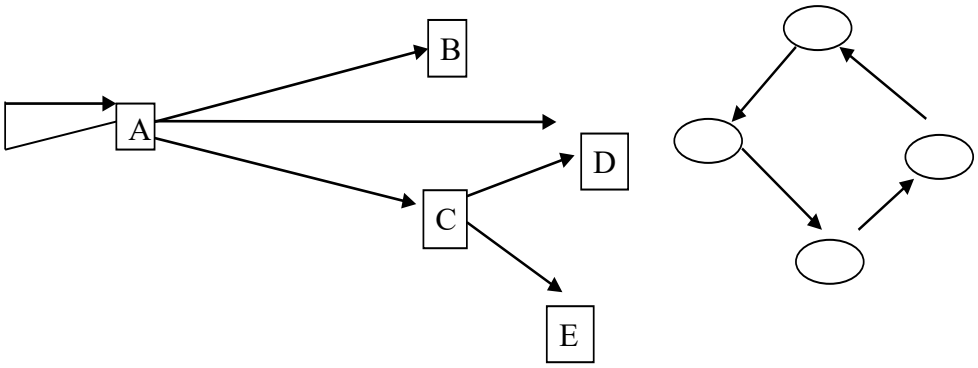
The set of vertices in the above graph is *{A, B, C, D, E}* and the set of edges are *{(A, B), (A, D), (A, C), (C, D), (C, E)}*

Adjacent Nodes

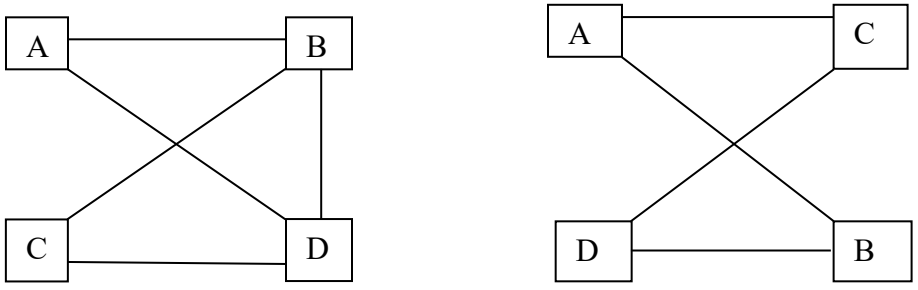
The definition of the graph *G = (V, E)* implies that to every edge of the graph *G*, we can associate a pair of nodes of the graph. If an edge $x \in E$ is thus associated with a pair of node *(u, v)* where $u, v \in V$, then we say that the edge *x* connects or joins the nodes *u* and *v*. Any two nodes which are connected by an edge in a graph are called *adjacent Nodes*.

Directed and undirected graph

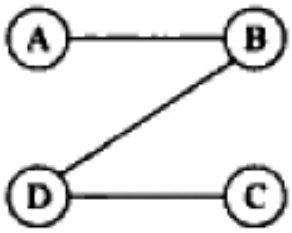
In a graph *G = (V, E)* an edge which is directed from one node to another is called a *directed edge*, while an edge which has no specific direction is called an *undirected edge.* A graph in which every edge is directed is called a *directed graph* or a *diagraph.* A graph in which every edge is undirected is called an *undirected graph.* If some of the edges are directed and some are undirected in a graph, then the graph is a *mixed graph.* A *directed graph* is also called as ***DAG***. The *directed graphs* are shown in the following figure:



Directed Graphs

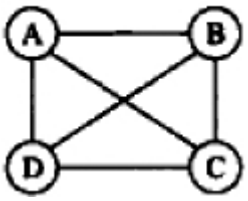


Undirected Graph


A
B
D
C

Connected Graph

A graph is called connected if there is a path from any vertex to any other vertex. A graph $G = (V, E)$ is connected if and only if there is a simple path between any two nodes in G. A graph G is said to be completed if every node u in G is adjacent to every other node v in G. A complete graph with n *nodes* has $n * \frac{(n-1)}{2}$ *edges*. The connected graph can show as:



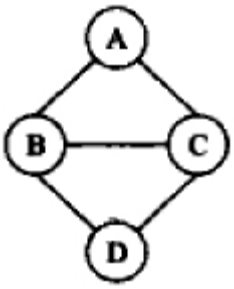
Connected Graph

Cycle

A path from a node to itself is called a cycle. Thus, a cycle is a path in which the initial and final vertices are same. Acyclic graph with all vertex connected is a tree.

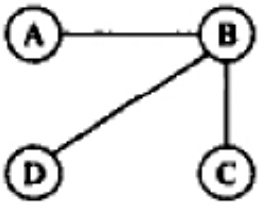
Example

Let us consider the following graph:



Cycle in the Graph

The path (A, B, C, A) or (A, C, D, B, A) are cycles of different lengths in the graph. If a graph contains a cycle, it is ***cyclic,*** otherwise it is ***acyclic.*** A tree is a graph but the graph does not need to be a tree.



Free tree or graph without cyclic

Loop

If an edge is having identical end points, then the edge is called a loop. Thus, an edge *e* is called a loop if it has identical end points, i.e. *e = (u, u)*.

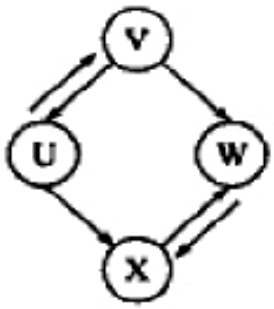
Degree, incidence, adjacency

A vertex ***v*** is **incident** to an edge *e* if *V* is one of the two vertices in the ordered pair of vertices that constitute *e*.

The ***degree*** of a vertex is the number of edges incident to it. The ***in-degree*** of a vertex ***V*** is the number of edges that have ***V*** as the head and the ***out-degree*** of a vertex ***V*** is the number of edges that have ***V*** as the tail.

Example

Let us consider the following graph



In this graph vertex ***V*** has *in-degree **1**, out-degree **2*** and *degree **3***. A ***V*** vertex is adjacent to vertex ***U*** if there is an edge from ***U*** to ***V***. If ***V*** is adjacent to ***U***, ***V*** is called a successor of ***U*** and ***U*** a predecessor of ***V***.

Isolated node

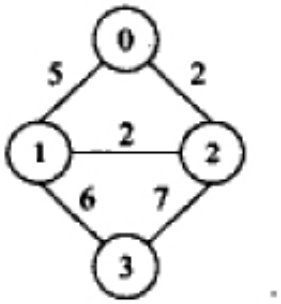
If degree of a node is zero i.e. if the node is not having any edges, then the node is called *isolated node.*

Complete Graph

A graph is called complete if all the nodes of the graph are adjacent to each other. A complete graph with n nodes will have n*(n-1)/2 edges.

Weighted Graph

A graph is said to be weighted if each edge in the graph is assigned a non-negative numerical value called the weight or cost of the edge. If an edge does not have any weight then the weight is considered as 1.



Weighted Graph

Multi-graph

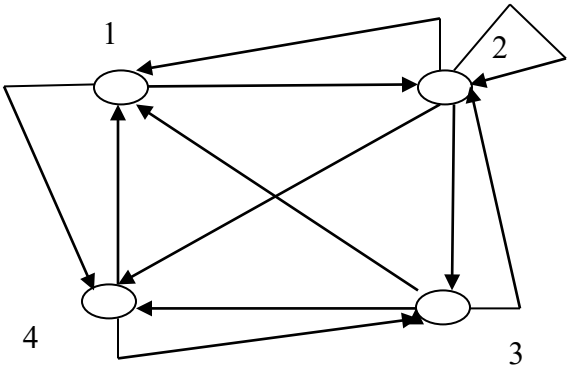
If a graph has two parallel paths to an edge or multiple edges along with loop is said to be *multi-graph*.

Sub Graph

A Graph ***G'*** is called a sub-graph of ***G = (V, E)*** if ***V'*** is a subset of ***V*** and ***E'*** is a subset of ***E***. For ***G***' to a sub-graph of ***G*** if all the edges and vertices of ***G'*** should be in ***G***.

Example

Consider the following graph and obtain simple and elementary paths form it.



In this graph some of the paths originating in node 2 and ending in node 4 are:

$P_1 = ((2, 4))$

$P_2 = ((2, 3), (3, 4))$

$P_3 = ((2, 1), (1, 4))$

$P_4 = ((2, 3), (3, 1), (1, 4))$

$P_5 = ((2, 3), (3, 2), (2, 4))$

$P_6 = ((2, 2), (2, 4))$

In this the paths P_1, P_2, P_3, and P_4 are *elementary paths* while paths P_5 and P_6 are simple but not elementary.

The following are some of the cycles in this graph:

$C_1 = ((2, 2))$

$C_2 = ((1, 2), (2, 1))$

$C_3 = ((2, 3), (3, 1), (1, 2))$

$C_4 = ((2, 1), (1, 4), (4, 3), (3, 2))$

9.3 GRAPH REPRESENTATION

A diagrammatic representation of a graph may have a limited usefulness. However, such a representation is not feasible when the number of nodes and edges in a graph is large. There are different methods for the alternative representation of graph. The basic three methods for the methods for the graph representation are as follows:

- Representation of the graph by using matrices.
- Representation of a graph by a list structure.
- Representation of a graph by a Adjacency lists and edge lists.

Matrix Representation of Graphs

Given a simple diagraph $G = (V, E)$, it is necessary to assume some kind or ordering of the nodes of a graph in the sense that a particular node is called a first node, a nother second node, and so on. A matrix representation of G depends upon the ordering of the nodes. Therefore, in the given diagraph G we have $V = \{v_1, v_2, \ldots\ldots v_n\}$ and the nodes are assumed to be ordered from v_1 *to* v_n. An n *X* n matrix A whose elements a_{ij} are given by:

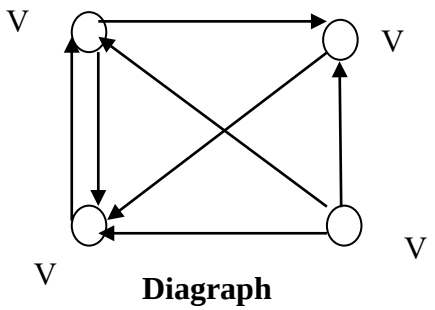
$$a_{ij} = \begin{cases} 1 if (v_i, v_j) \in E \\ 0 otherwise \end{cases}$$

This is called the *adjacency matrix* of the graph G.

Any element of the *adjacency matrix* is either *0* or *1*. Any matrix whose elements are *0* or *1* is called a *bit matrix* or a *Boolean matrix*. Hence the i^{th} row in the *adjacency matrix* is determined by the edges which originate in the node v_i. The number of elements in the i^{th} row whose value is *1* is equal to the out-degree of the node v_i. Similarly, the number of elements whose value is *1* in a column, say the j^{th} column, is equal to the in-degree of the node v_j. An *adjacency matrix* completely defines a simple diagraph. If two diagraphs are such that the *adjacency matrix* of one can be obtained from the *adjacency matrix* of the other by interchanging some of the rows and the corresponding columns, then the diagraphs are equivalent.

Example

Obtain the *adjacency matrix* for the following diagraph.



Diagraph

The corresponding *adjacency matrix* for this diagraph is as follows:

$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

Adjacency Matrix

Now we consider the powers of an adjacency matrix. Naturally, an entry of *1* in the i^{th} row and j^{th} column of *A* shows the existence of an edge (v_i, v_j), that is, a path of length *1* from v_i to v_j. Let us denote the elements of A^2 by $a_{ij}^{(2)}$. Then:

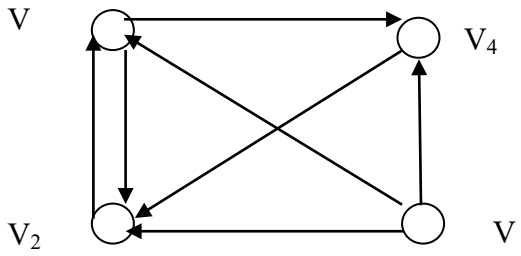
$$a_{ij}^{(2)} = \sum_{k=1}^{n} a_{ik} a_{kj}$$

So that for any fixed *k*, $a_{ik} a_{kj} = 1$ if and only if both a_{ik} and a_{kj} equal 1; i.e. (v_i, v_k) and (v_k, v_j) are edges of the graph. For each such *k* we get a contribution of *1* in the sum. Now (v_i, v_k) and (v_k, v_j) imply that there is a path from v_i to v_j of length 2. Therefore, a_{ij}^2 is equal to the number of different paths of exactly length *2* from v_i to v_j. Similarly, the diagonal element a_{ij}^2 shows the number of cycles of length *2* at the node for v_i for *i=1, 2,….,n*. By a similar argument, one can show that the element in the i^{th} row and j^{th} column of A^3 gives the number of paths of exactly length *3* from v_i to v_j. In general, the following statement can be shown:

Let *A* be the *adjacency matrix* of the digraph *G*. The element in the i^{th} row and j^{th} column of A^n *(n>1)* is equal to the number of paths of length *n* from the i^{th} node and j^{th} node.

Example

Consider the following diagraph:



Diagraph

The corresponding *matrices* A^2, A^3 *and* A^4 for this diagraph can represent as:

$$A^2 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 2 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \qquad A^3 = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 2 & 2 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 2 & 3 & 0 & 2 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

9.4 PATH MATRIX OR REACH-ABILITY MATRIX

Suppose $G = (V, E)$ is a simple digraph with n nodes and the nodes of G are being ordered and are called $v_1, v_2, v_3, \ldots, v_n$. An $n \times n$ matrix P is a Path matrix of the diagraph G if:

$$P_{ij} = \begin{cases} 1, \text{ if there is a path between } v_i \text{ and } v_j \\ 0, \text{ otherwise} \end{cases}$$

The *path matrix* only shows the presence or absence of at least one path between a pair of points and also the presence or absence of a cycle at any node. It is easily shown that in a simple diagraph with *n* nodes, the length of an elementary path or cycle does not exceed *n*. Also, for a path between any two nodes, one can obtain an elementary path by deleting certain parts

of the path that are cycles. Similarly (for cycles), we can always obtain an elementary cycle from a given cycle. If we are interested in determining whether there exists a path from v_i to v_j, all we need to examine are the elementary paths of length less than or equal to *n-1*. In the case where $v_i = v_j$ and the path is a cycle, we need to examine all possible elementary cycles of length less than or equal to *n*. Such cycles or paths are easily determined from the matrix B_n where;

$$B_n = A + A^2 + A^3 + ... + A^n$$

The element in the i^{th} row and j^{th} column of B_n shows the number of paths of length n or less which exist from v_i to v_j. If this element is non-zero, then it is we need to know the existence of a path, and not the number of paths between any two nodes. In any case, then matrix B_n furnishes the required information about reach-ability of any node of the graph from any other node.

Therefore, the path matrix can be calculated from the matrix B_n by choosing $p_{ij} = 1$ if the element in the i^{th} row and the j^{th} column of B_n is non-zero, and $p_{ij} = 0$ otherwise.

Example

Now we apply this method of calculating the path matrix to the given diagraph in the above mentioned examples. The *adjacency matrix A* and the powers A^2, A^3 *and* A^4 have already been calculated in the previous example. We thus have B_4 and the path matrix *P* given by:

$$B_4 = \begin{bmatrix} 3 & 5 & 0 & 3 \\ 3 & 3 & 0 & 2 \\ 6 & 8 & 0 & 5 \\ 2 & 3 & 0 & 1 \end{bmatrix} \quad \text{and} \quad P = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

The method of obtaining the path matrix of a simple diagraph can easily be computed by using the following *WARSHALL* algorithm

Algorithm *WARSHALL* [Given the adjacency matrix A, this algorithm produces the path matrix P.]

1. [Initialize]
   $P = A$
2. [perform a Pass]
   Repeat through step 4 for k = 1, 2,…..,n
3. [Process rows]
   Repeat step 4 for I = 1, 2,……,n
4. [Process columns]
   Repeat for j = 1, 2,….,n
   $P_{ij} = P_{ij} \vee (P_{ik} \wedge P_{kj})$
5. [Finished]
   Exit

9.5 LINKED REPRESENTATION OF THE GRAPH

The Matrix representation of graph does not keeps track of the information related to the nodes. Hence a linked representation is used to represent a graph called adjacency structure. An adjacency list is a listing for each node of all edges connecting it to adjacent nodes. For a graph $G = (V, E)$, a list is formed for each element x of V, containing all nodes y such that (x, y) is an element of E. The manipulation of such structure is also known as the *list Processing*. The adjacency structure of the graph maintains two lists called *node list* and *edge list*.

Node List

Each node in the node list will correspond to a node in the graph and will have three field. They are the information of the node called INFO, Pointer to the next node of the list called NEXT, a pointer to the edge list called ADJ.

Edge List

Each element of the edge list will correspond to an edge of the graph and will give two fields. They are DEST contains the address of the destination node and LINK contains the address of the next node of the edge list.

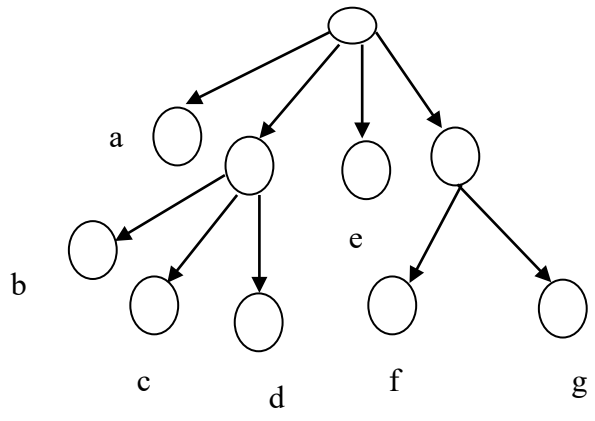
Representation

In the context of list processing, we define a list to be any finite sequence of zero or more *atoms or lists.* Here an atom is taken to be any object that is distinguishable from a list by treating the atom as structurally indivisible. If we enclose lists within parentheses and separate elements of lists by commas, then the following can be considered to be lists:

(i) *(a, (b, c, d), e, (f, g))*
(ii) *()*
(iii) *((a))*

The first element contains four elements, namely, the atom *a,* the list *(b, c, d)* which contains the atoms *b, c* and *d*, the atom *e*, and the list *(f, g)* whose elements are the atoms *f* and *g*. The second has no elements, but the null list is still a valid list. The third list has one element, the list *(a)*, which in turn contains the atom *a*. A graphic representation of these can show as:

(i) *(a, (b, c, d), e, (f, g))*



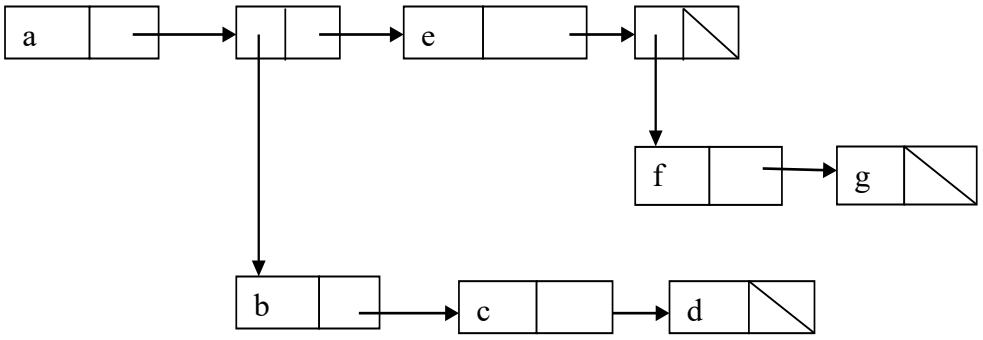
(ii) ( )

(iii) ((a))

Another notation which is often used to illustrate lists is similar to that used in the linked representation of trees. Each element of a list is indicated by a box and a pointer indicates whether the boxes are members of the same list or member of the sub lists. Each box is separate into two parts. The second part of an element contains a pointer to the next element in the same list or a null pointer to mark the end of the list. This representation contains the two types of links: (a) Horizontal pointer and (b) vertical pointer.

The horizontal pointer represents the relation of physical adjacency in a list. The vertical pointer specifies the non-atomic element or hierarchical relationship in a list. Now we consider this linked list representation for our given list as :

(i)

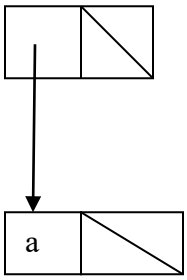


(a, (b, c, d), e, (f, g))

(ii) $\Longrightarrow$ (null Pointer)

( )

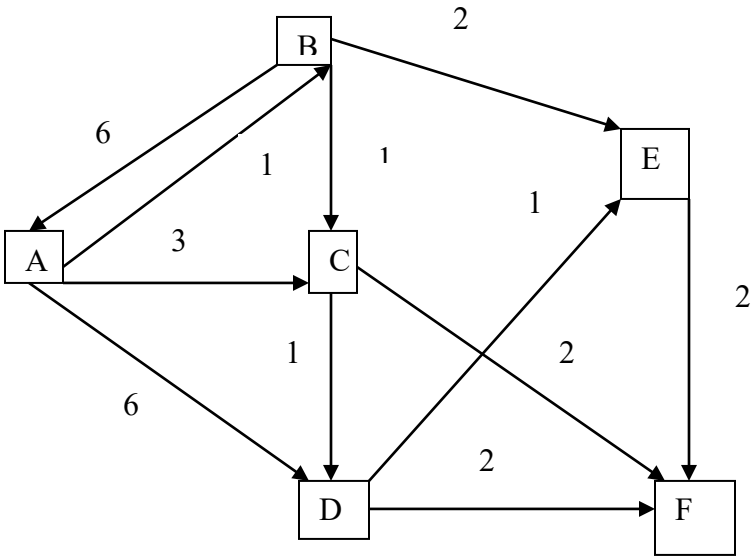
(iii)



((a))

Example

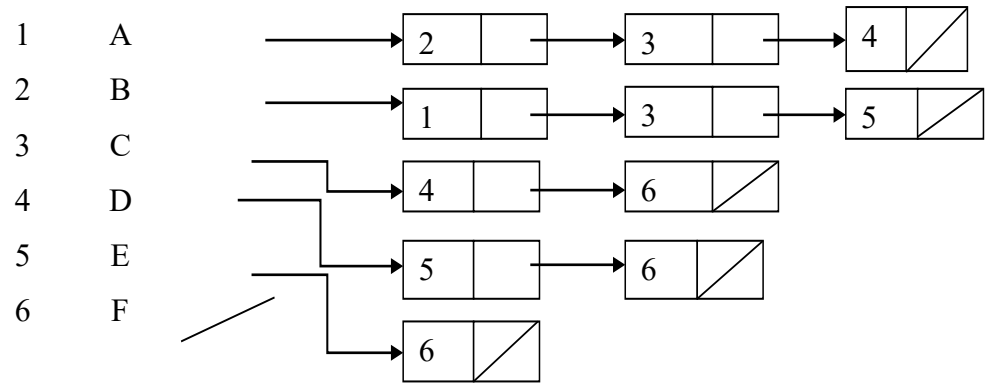
Consider the following graph and represent it with adjacency list edge list representation.



The Adjacency list representation of this graph is as:

a	b c d
b	a c e
c	d f
d	e f
e	F
f	

The edge list of this graph is as follows:



The list is given as:

(a, b), (a, c), (a, d), (b, a),

(b, c), (b, e), (c, d), (c, f)

(d, e), (d, f), (e, f)

9.6 GRAPH TRAVERSAL

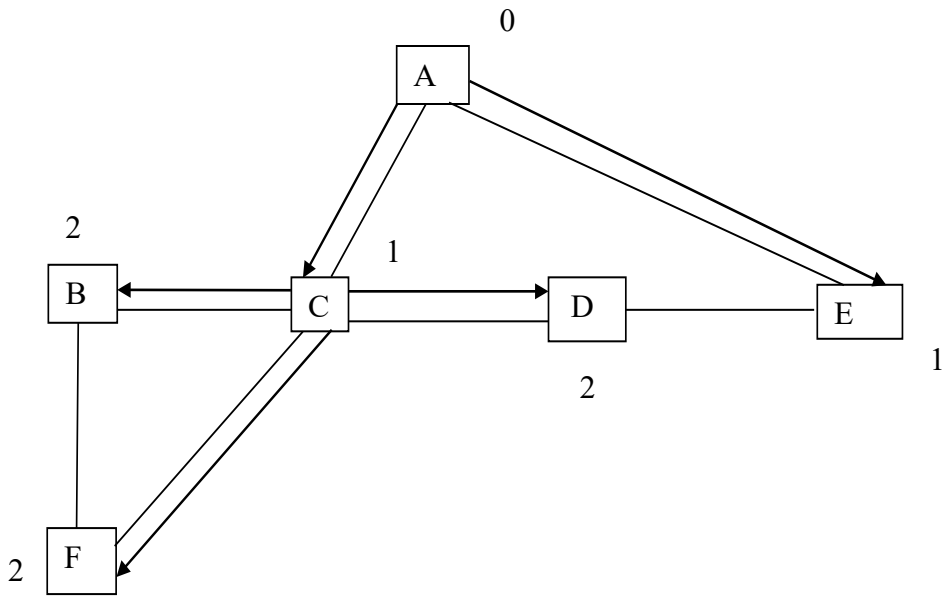
Traversing a graph means visiting all the vertices in a graph exactly one. Normally we use the following two basic searching methods for traversal of the graph:

- Breadth First Search
- Depth First Search

Breadth First Searching / Level order traversal

In general, breadth first search (BFS) can use to find the shortest distance between some starting node and the remaining nodes of the graph. This shortest distance is the minimum number of edges traversed in order to travel from the start node to the specific node being examined. In this searching we start from a node *v*, this distance is calculated by examining all incident edges to node *v*, and then moving on to an adjacent node *w* and repeating the process. The traversal continues until all nodes in the graph have been examined. A queue is maintained in this technique to maintain the list of incident edges and marked nodes. It is more appropriate for a digraph.

We can consider this searching strategy from the following diagraph:



Using the BFS searching on this graph, the in-directed traversal results, assuming node *A* is the start position and each edge is assigned a value of one. The shortest distance from the start is given by the number associated with each node . All nodes adjacent to the current node are numbered before the search is continued. This ensures every node will be examined at least once. The efficiency of a BFS algorithm depends on the method used to represent the graph. The adjacency list representation is suitable for this algorithm, since finding the incident edges to the current node involves simply traversing a linked list, whereas an adjacency matrix would require searching only particular row is traversed and that too only one time. The representation of the node to implement this algorithm is as follows:

REAC	NODENO	DAT	DIST	LISTPT

Node table directory structure

DESTI	EDGEP

Edge Structure

Here:

REACH specifies whether a node has been reached in the traversal and its initial value is *false.*

NODENO identifies the node number.

DATA contains the information pertaining to this node.

DIST is the variable which will contain the distance from the start node.

LISTPTR is a pointer to a list of adjacent edges for the node.

DESTIN contains the number of the terminal node for this edge.

EDGEPTR points to the next edge in the list

Now, we represent the algorithm which calculates BFS distances using two sub-procedures i.e. Q INSERT and Q DELETE (we have already discussed them in unit six (06)). QINSERT enters a value onto the rear of a queue, in this case a node whose incident edges have not yet been examined. The procedure has two parameters, the queue name and the value to be inserted. QDELETE removes a value from the front of a queue specified, placing it in INDEX. In the algorithm, this value is the next node which will be processed. Now we write the algorithm formally as:

Procedure BFS (INDEX): [This algorithm generates the shortest path for each node using a breadth first search (*BFS). INDEX* denotes the current node being processed and *LINK* points to the edge being examined. It is assumed that the *REACH* field has been yet set to *false* when the structure was created. *QUEUE* denotes the name of the queue.]

1. [initialize the first node's *DIST* number and place node in queue]

   REACH [INDEX] = TRUE;

   DIST [INDEX]=0 ;

   QINSERT(QUEUE, INDEX);

2. [Repeat until all nodes have been examined]

   Repeat thru step 5 while queue is not empty.

3. [Remove current node to be examined from queue]

   QDELETE (QUEUE, INDEX);

4. [Find all unlabeled nodes adjacent to current node]

   LINK = LISTPTR [INDEX];

   Repeat step 5 while LINK ≠NULL

5. [If this is an unvisited node, label it and add it to the queue]

   If not REACH [DESTIN (LINK)]

   Then DIST [DESTIN (LINK)]= DIST[INDEX] + 1;

   REACH [DESTIN(LINK)] = TRUE;

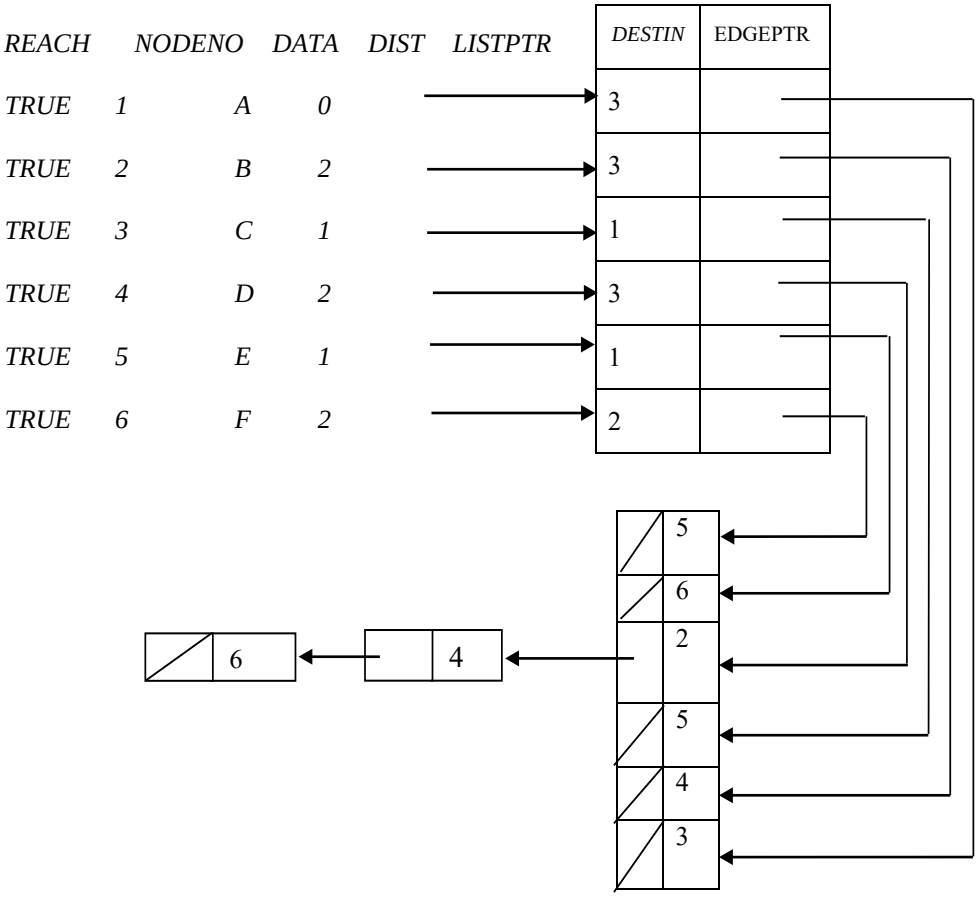
   QINSERT(QUEUE, DESTIN(LINK))

   LINK = EDGEPTR(LINK) */* move down edge list */*

6. [Finished]

   Return

The storage representation for the above given graph can be shown as after the result of this algorithm:



Storage representation of given graph

Check your progress:

A graph is connected if for every two nodes X and Y, there is either a path from Y to X or from X to Y. modify Algorithm BFS to determine if a graph is connected.

Depth First Search

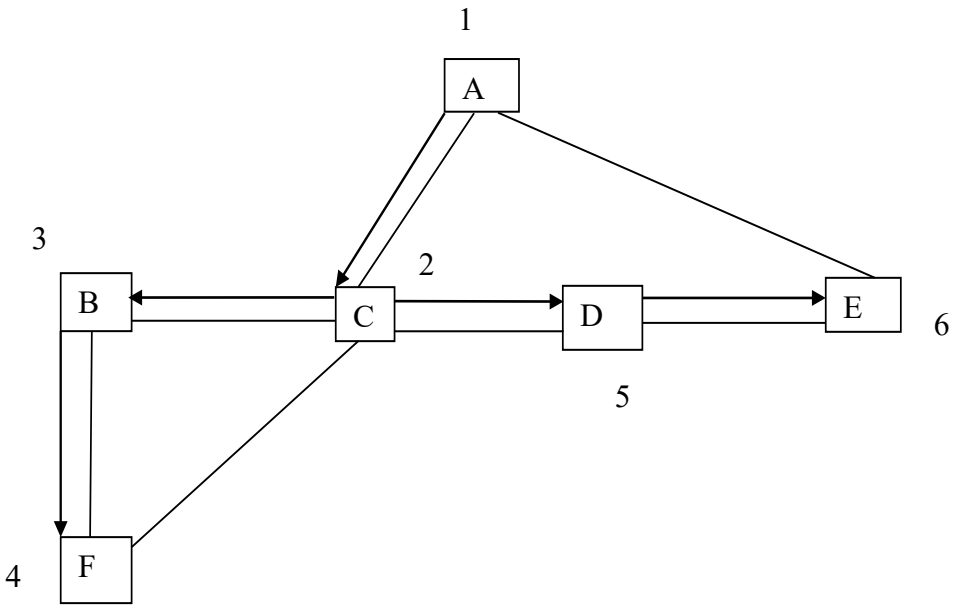
A depth first search (DFS) of an arbitrary graph can be used to perform a traversal of a general graph. As each new node is encountered, it is marked with *True* indicating the order in which the node was encountered to show that the node has been visited. The DFS strategy is as follows:

A node *S* is picked as a start node and marked. An unmarked adjacent node to *S* is now selected and marked, becoming the new start node ,

possibly leaving the original start node with unexplored edges for the present. The search continues in the graph until the current path ends at a node with out-degree *zero* or at a node with all adjacent nodes already marked. Then the search returns to the last node which still has unmarked adjacent nodes and continues marking until all nodes are marked.

Example

Consider the following graph:



Depth First Search Traversal

Therefore in this strategy result of the traversal indicated by the arrows, assuming each edge has been assigned a value of one. Starting at node A, the search numbers all nodes down until node *F*, where all adjacent nodes have already been marked. This strategy returns to node C, which still has an unlabeled adjacent node D. After node D and E are labeled, all nodes are numbered and the search is complete. Thus the orders in which the nodes will traverse are: *A C B F D E*.

Since adjacent nodes are needed during the traversal, the most efficient representation is adjacency list. Therefore, the same data structure as used in BFS will use here with the change that the DIST variable in the node table directory to DFN (Depth first search number) as:

REAC	NODENO	DAT	DFN	LISTPT

Node table directory structure

DESTI	EDGEP

Edge Structure

It is clear from the example that the stack is required to implement the DFS traversing. The formal algorithm for DFS is as follows:

Procedure DFS (INDEX, COUNT): [This recursive algorithm calculates the depth first search numbers for a graph. *INDEX* denotes the current node being processed and *LINK* points to the edge being examined. *COUNT* is used to keep track of the current *DFN* number and is initially set to zero outside the procedure. Finally, it is assumed the *DFN* filed was initialized to zero when the adjacency list structure was created]

1. [Update the depth first search number, set and mark current node]

   COUNT = COUNT + 1;

   DFN [INDEX] = COUNT;

   REACH [INDEX] = TRUE;

2. [Set up loop to examine each neighbor of current node]

   LINK = LISTPTR [INDEX];

   Repeat step 3 while LINK ≠NULL.

3. [If node has not been marked, label it and make recursive call]

   If not REACH [DESTIN (LINK)]

   Then DFS (DESTIN (LINK), COUNT);

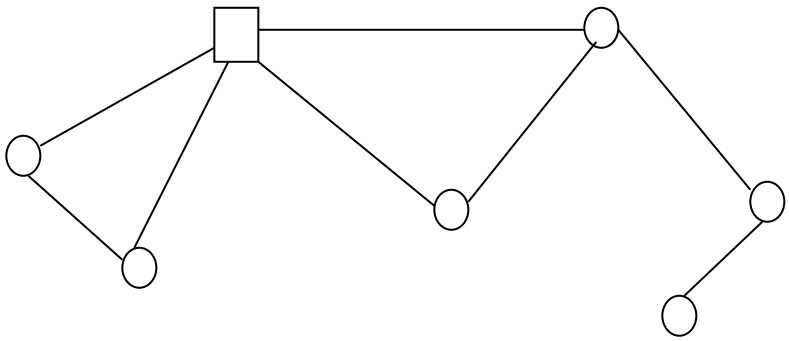
   LINK = EDGEPTR(LINK) */* Examine next adjacent node */*

4. [Finished]

   Return

Check your progress :

Produce the input structure and DFN number for the following graph.



9.7 SPANNING TREE

A spanning tree of a graph is an undirected tree consisting of only those edges necessary to connect all the nodes in the original graph. A spanning tree has the properties that for any pair of nodes there exists only one path between them, and the insertion of any edge to a spanning tree forms a unique cycle. Those edges left out of the spanning tree were present in the original graph connect paths together in the tree. When determine the cost of a spanning tree of a weighted graph, the cost is simply the sum of the weights of the tree's edges. A minimal cost spanning tree is formed when the edges are picked to minimize the total cost.

We may define the spanning tree alternatively as follows also:

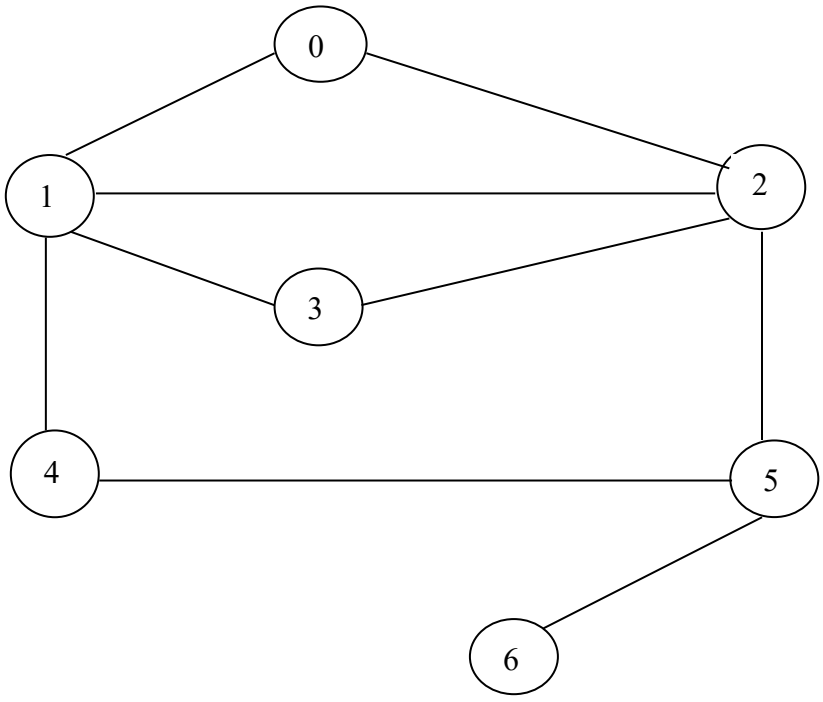
"If G is a weighted graph and T_1, T_2 are two spanning trees of G than the sum of weights of all edges in T_1 may be different from that of T_2. A spanning tree T of G where the sum of weights of all edges in T is minimum is called the ***minimal cost spanning tree*** or ***minimal spanning tree*** of G.

Mathematical interpretation of the spanning tree can understand as follows:

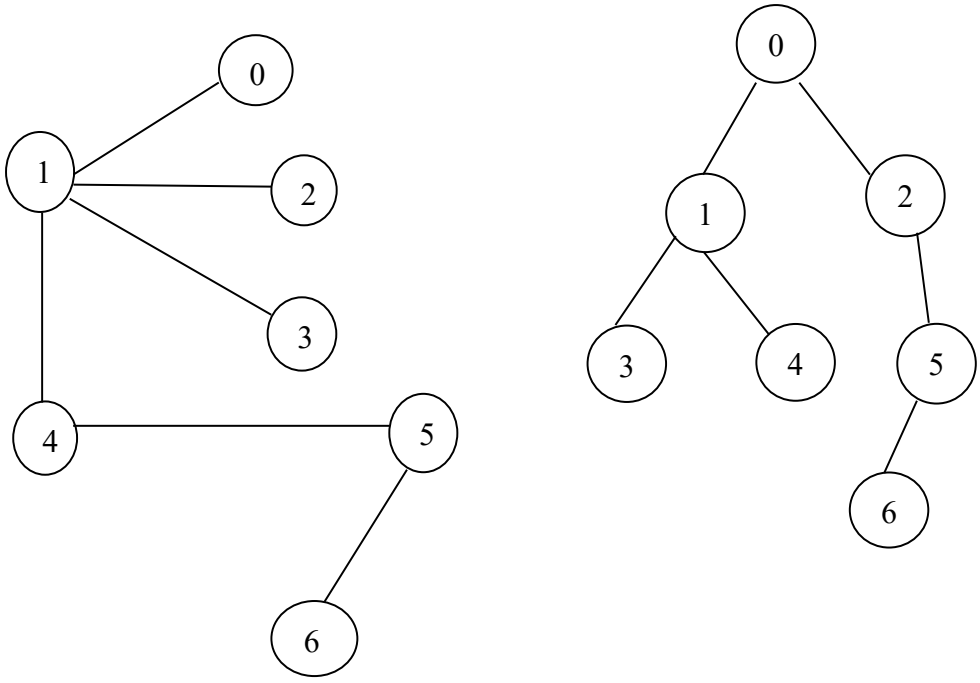
A sub-graph of a Graph ***G = (V, E)*** which is tree containing all vertices of ***V*** is called a spanning tree of ***G***. If ***G*** is not connected then there is no spanning tree of ***G***.

Example

Now consider the graph ***G*** given as follows:



There are two spanning trees of this graph can construct. These spanning trees are as follows:



Spanning Trees

9.8 ALGORITHM FOR COMPUTING MINIMAL SPANNING TREE

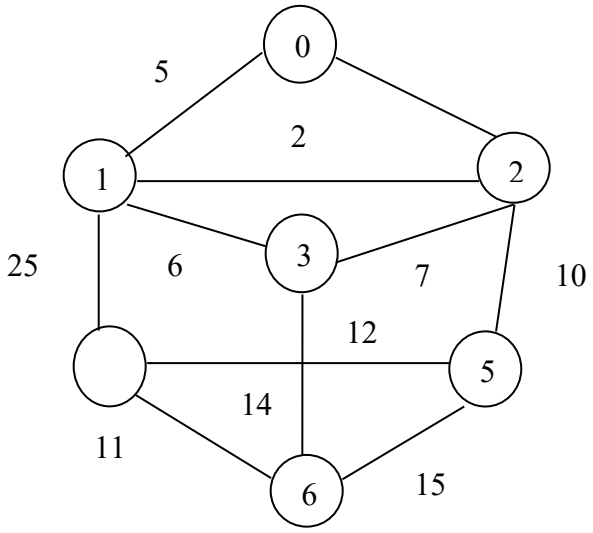
There are two popular algorithms to calculate minimal cost spanning tree of a weighted undirected graph:

(a) Kruskal's Algorithm

Kruskal's algorithm functions on a list of edges of a graph where the list is arranged in order of weight of the edges. It is a greedy algorithm. In each step it chooses the highest edge that does not create a cycle. We start with n trees, each consisting of a single vertex. Then trees are joined until in the end, we have a single tree. Thus in each step an edge is selected and added if the incorporation does not from a cycle. The edge which is selected, it is deleted from the list of edges. The algorithm continues until **(n-1)** edges are added to the list of edges exhausted. When the algorithm ends after adding **(n-1)** edges, a minimum spanning tree is produced.

Example

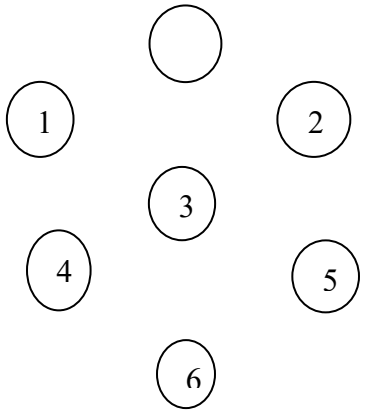
Consider the following graph. There are eleven edges. An edge connecting the vertices 'i' and 'j' may be represented by a **tuple (i, j)**



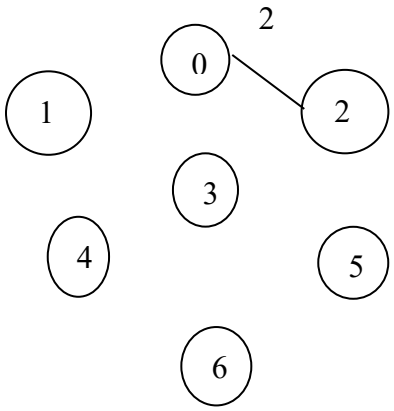
The list of edges of the graph sorted in non-descending order may be given by:

{(0,2), (1, 2), (0, 1), (1, 3), (2, 3), (2, 5), (4, 6), (4, 5), (3, 6), (5, 6), (1, 4)}

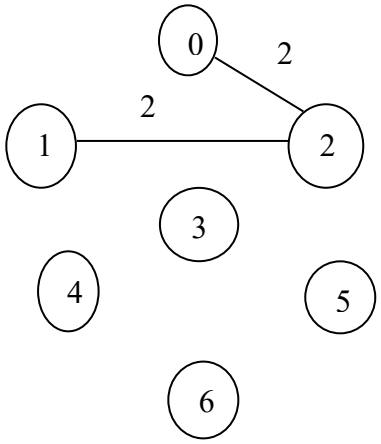
The initial tree we can show as:



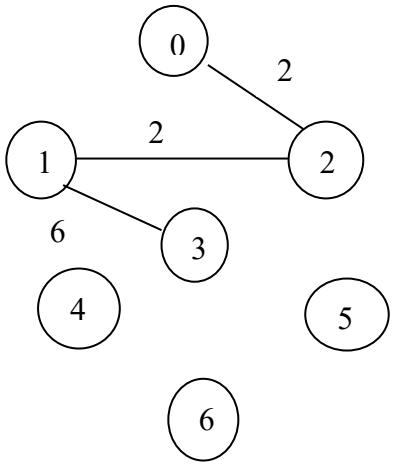
The first edge (0,2) from the list of edges is taken into account. Since the addition of this edge does not form a cycle. It can be added to change the partial minimum spanning tree as show below:



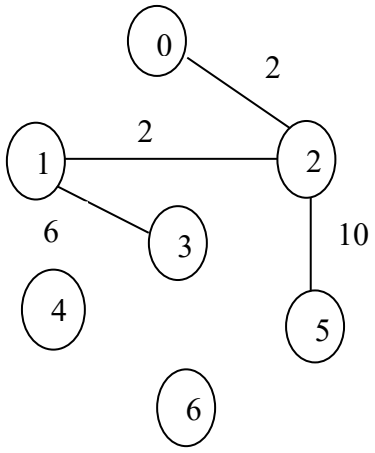
Now, (1, 2) is the next edge to be considered. Inclusion of this edge does not result in a cycle and hence it will be added to the partially formed minimum spanning tree, this can be shown as:



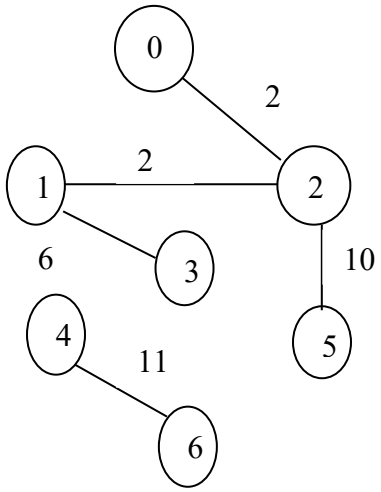
The next edge (0, 1) is not added to the partially formed tree as it forms a cycle. The next edge (1, 3) is now taken into account. Since inclusion of this edge does not form a cycle, this edge can be included and the partial minimum spanning tree is created, it can be shown as:



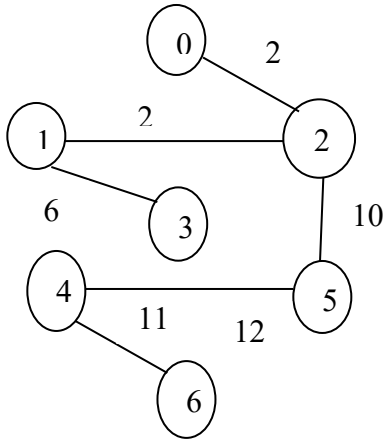
If the next edge (2, 3) is added then a cycle 2-1-3-2 is formed. Therefore, this edge is not added to the partially formed tree. The next edge (2, 5) is selected. Since inclusion of this edge does not form a cycle, this edge can be added and the partial minimal spanning tree is formed as:



Next the edge (4, 6) is chosen. This edge can be added. This can be shown as:



The next edge in the list (4, 5) is added to complete the minimal spanning tree as:



Now we formally define the kruskal's Minimum spanning tree algorithm as:

Function Kruskal [This algorithm accepts a connected weighted undirected graph ***G = (V, E, w)*** as the input and produces a minimum spanning tree ***T*** of ***G*** as output]

1. *Sort the edges by increasing weight*
2. *For each vertex v Є V do*
3. *Make-set (v)*
4. $E_T = \Phi$
5. *For each edge e = {u, v} Є E, in order by increasing weight do*
6. *If Find (u) ≠Find (v) then*
7. $E_T = E_T \bigcup \{e\}$
8. *Union (u, v)*
9. *Return (V, E_T)*

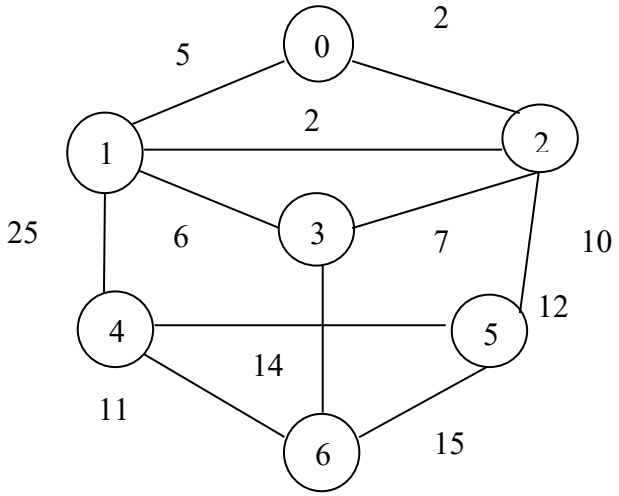
(b) Prim's Algorithm

Prim's algorithm also works in a greedy manner but it does not grows everal components. It just extends on e component by successively adding new nodes. In this algorithm we store all vertices in a priority queue. As long as a vertex *y* is not adjacent to a vertex of the spanning tree selected so far, its key is set to ∞. If it becomes adjacent to some vertex say *x,* then its key becomes the weight of *w({x, y}).* If it becomes a adjacent to a new vertex, then we only set the key to the new weight if this will decrease the key. Thus, *Key [x]* is the cost of adding *x* to the tree grown so far.

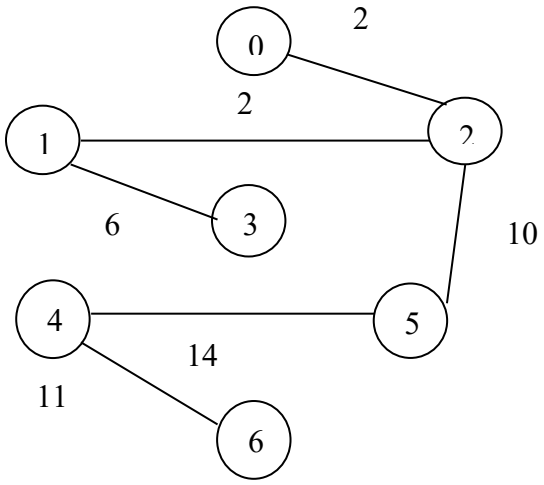
Therefore we can say that the Prim's algorithm starts with any arbitrary vertex as the partial minimal spanning tree ***'T'***. In each iteration of the algorithm one edge say ***(u, v)*** is added to the partial tree ***'T'*** so that exactly one end of this edge belongs to the set of vertices in ***'T'***. Hence, of all such possible edges, the edge having the least cost is selected. The algorithm continues to add ***(n-1)*** edges.

Example

Consider the following graph:



Now we start with the vertex 0 and construct the minimal spanning tree from prim's algorithm as:



Now we formally define the Prim's Minimum spanning tree algorithm as:

Function Prim's [This algorithm accepts a connected weighted undirected graph ***G = (V, E, w)*** as the input and produces a minimum spanning tree ***T*** of ***G*** as output]

1. *Choose an arbitrary root r Є V.*
2. *Key [r] = 0; key [v] = ∞ for all v Є V expect r.*
3. *Let Q be a min-priority queue filled with the vertices in V.*
4. *P [r] = NULL*
5. *While Q is not empty do*
6. *X = Extract-min (Q)*
7. *If p [x] ≠ NULL then*

8. *Et = $E_T \bigcup \{p[x], x\}$*

9. *For each vertex y adjacent to x do*

10. *If y $\in$ Q and w({x, y}) < Key [y] then*

11. *P[y] = z;*

12. *Key [y] = w({x, y})*

13. *Return (V, E_T)*

9.9 SHORTEST PATH ALGORITHMS

Suppose we are given a directed graph ***G*** in which every edge has a weight attached and our problem is to obtain the path which has the shortest length between source and destination nodes or the length of shortest path between the nodes. Thus we can say that let ***G = (V, E, w)*** be an edge-weighted directed graph and let *s, t $\in$ V*. Let *P = (v_0,…..,v_l)* be the path from *s* to *t*, that is *v_0= s, v_l=t*. The issue is here to compute the shortest path from *s to t*. The first point which we have to investigate is that is that whether the shortest walk from *s* to *t* is always a path. This is certainly the case if all weights are ***non-negative***. Consider a walk from *s* to *t* that is not a path. Then this walk contains a cycle *C*. If the weight of *C* is non-negative, then we can remove this cycle and get a new walk whose weight is equal to *w(C) = 0* or shorter than if *w(C)>0* the weight of the original walk. If the weight of *C* is negative, then the walk gets shorter if we go through *C* once more. In this case, there is no shortest walk from *s to t*. Thus, if on every walk from *s* to *t* there is no negative cycle, then the weight of shortest walk from *s* to *t* is well – defined and is always attained by a path. If there is a walk with a negative cycle from *s* to *t*, then we set weight to -∞.

Now we explore the general algorithm of determining the shortest path from the given weighted diagraph. This algorithm is known as ***WARSHALL*** algorithm. This algorithm is used to obtain a matrix which gives the lengths of shortest paths between the nodes. The algorithm is defined as follows:

Algorithm MINIMAL [Given the adjacency matrix, B, in which the zero elements are replaced by infinity or some very large number, the matrix C produced by this algorithm shows the minimum lengths of paths between the nodes.]

1. *[Initialize]*

   C = B;

2. *[Perform a pass]*

   Repeat thru step 4 for k = 1, 2,,….,n

3. *[process row]*

4. *For j = 1 to n do*

$$C_{ij} = MIN(C_{ij}, c_{ik} + c_{kj})$$

5. *[Finished]*

Exit.

Example

Let us consider the following adjacency matrix of any diagraph:

$$P = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Now the matrix Q can be obtained by using the ***WARSHALL'S*** algorithm as:

$$Q = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Now we define another algorithm for determining the shortest path. This algorithm is known as the ***Dijkstra Algorithm.***

In this algorithm we begin by examining how to alter *D (w)*. In solving the un-weighted shortest-path problem, if *D (w)* = ∞, we set *D (w) = D (v) +1* because we lower the value of *D (w)* if vertex offers a shorter path to w. The algorithm ensures that we need to alter D (w) only once. We add *1* to *D (v)* because the length of the path to *w* is *1* more than the length of the path to *v*. If we apply this logic to the weighted case, we should set *D (w) = D (v) + C v, w* if this new value of *D (w)* is better than the original value. However, we are no l onger guaranteed that *D (w)* is altered only once. Consequently, *D (w)* should be altered if its current value is larger than *D (v) + C v, w*. The algorithm decides whether *v* should be used on the path to *w*. The original cost *D (w)* is the cost without using *v*; the cost *D (v) + C v, w* is the cheapest path using v. Problem is to find a path from one vertex ***V*** to another ***W*** such that the sum of the weights on the path is as small as possible. We call such path a ***shortest path***. It is important to note that length of a path in a weighted graph is defined to be sum of costs or weights of all edges in that path. In general there could be more than one path between a pair of specified vertices. Thus the shortest path between two vertices may not be unique.

This algorithm assumes that all weights of the digraph are non-negative. This algorithm considers the input as the edge-weighted directed graph ***G***

= *(V, E, w)*, *w (e)* ≥ *0* for all *e*, source *s* Є V. The output of this algorithm is the shortest path tree. The algorithm is defined as follows:

Algorithm Dijkstra

1. *Initialize d [s] = 0, d [v] = ∞, for al v ≠ s and p [v] = NULL for all v.*
2. *Let Q be a min-priority queue filled with all vertices from V using d [v] as keys.*
3. *While Q is not-empty do*
4. *x = min (Q)*
5. *for each y with (x, y) Є E do*

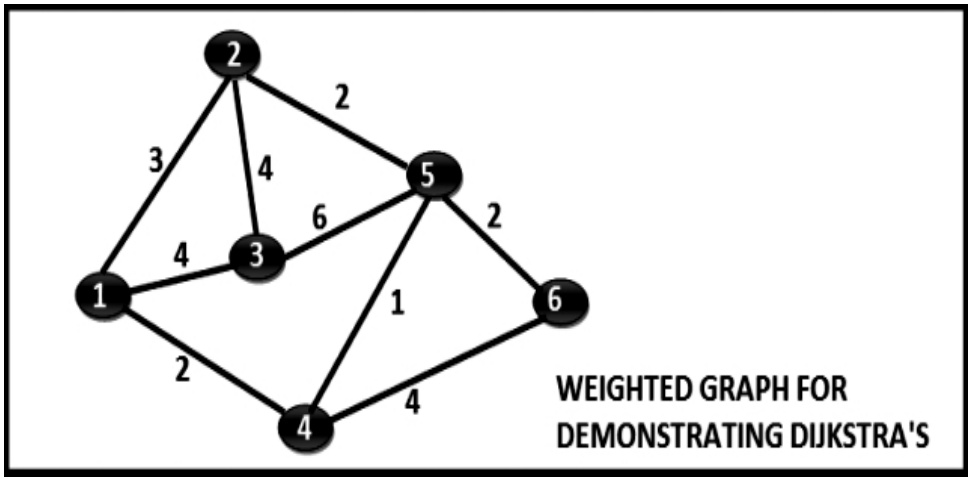
   if d [y] > d[x] + w((x, y)) then

   d [y] = d [x] + w ((x, y))

   p [y] = x;
6. *exit*

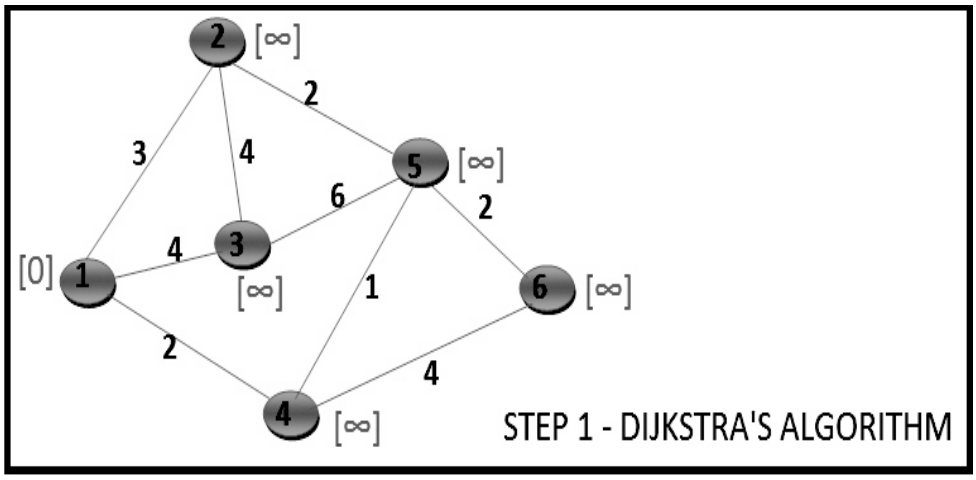
Example

The following implementation is showing the stages of the ***Dijkstra's algorithm*** as follows for the given graph:



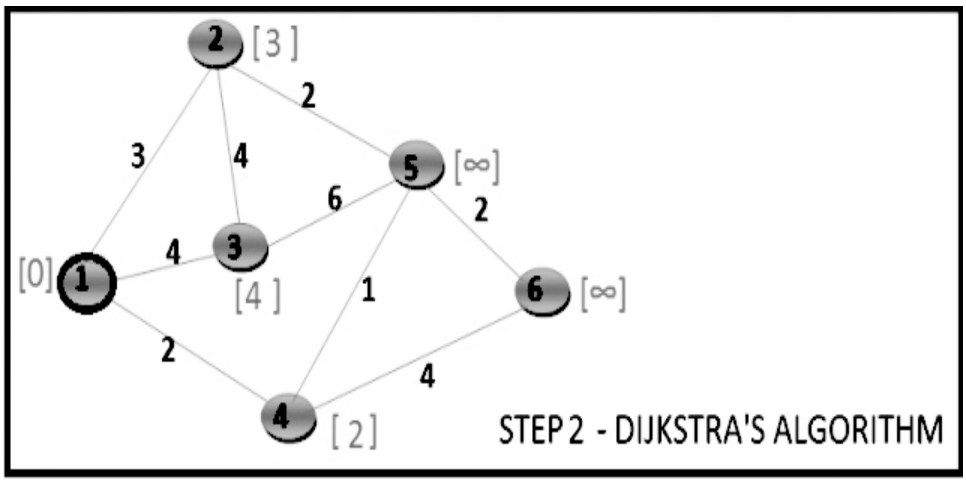
Step 1:

Mark Vertex 1 as the source vertex. Assign a cost zero to Vertex 1 and (infinite to all other vertices). The state is as follows:



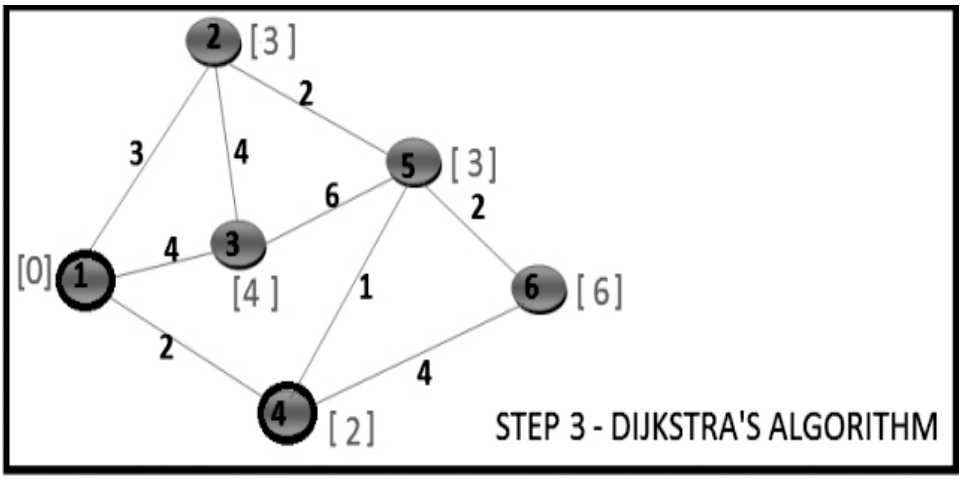
Step 2:

For each of the unvisited neighbors (Vertex 2, Vertex 3 and Vertex 4) calculate the minimum cost as min (current cost of vertex under consideration, sum of cost of vertex 1 and connecting edge). Mark Vertex 1 as visited, in the diagram we border it black. The new state would be as follows:



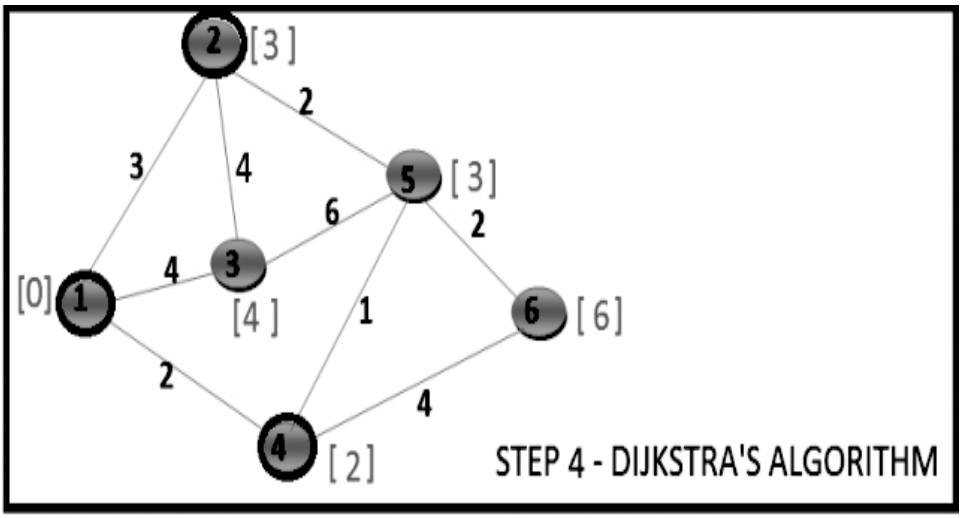
Step 3:

Choose the unvisited vertex with minimum cost (vertex 4) and consider all its unvisited neighbors (Vertex 5 and Vertex 6) and calculate the minimum cost for both of them. The state is as follows:



Step 4:

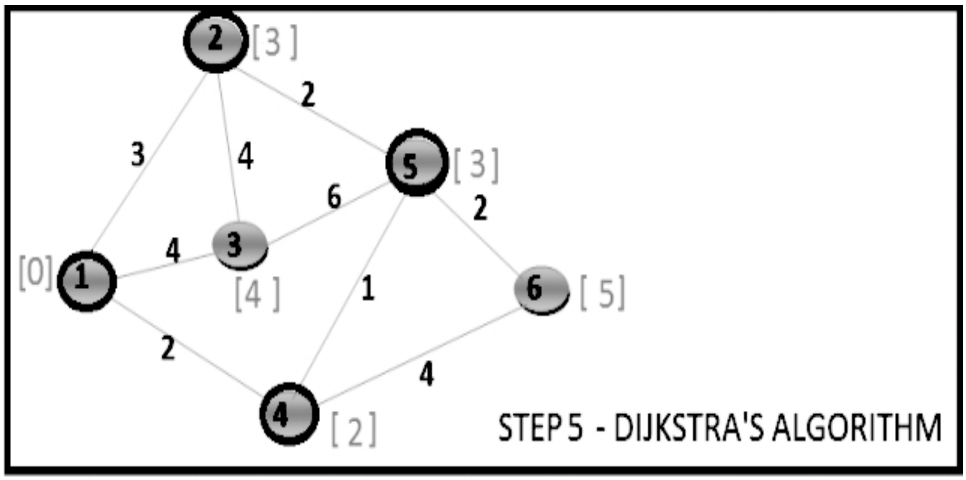
Choose the unvisited vertex with minimum cost (vertex 2 or vertex 5, here we choose vertex 2) and consider all its unvisited neighbors (Vertex 3 and Vertex 5) and calculate the minimum cost for both of them. Now, the current cost of Vertex 3 is [4] and the sum of (cost of Vertex 2 + cost of edge (2,3)) is 3 + 4 = [7]. Minimum of 4, 7 is 4. Hence the cost of vertex 3 won't change. By the same argument the cost of vertex 5 will not change. We just mark the vertex 2 as visited, all the costs remain same. The state is as follows:



Step 5:

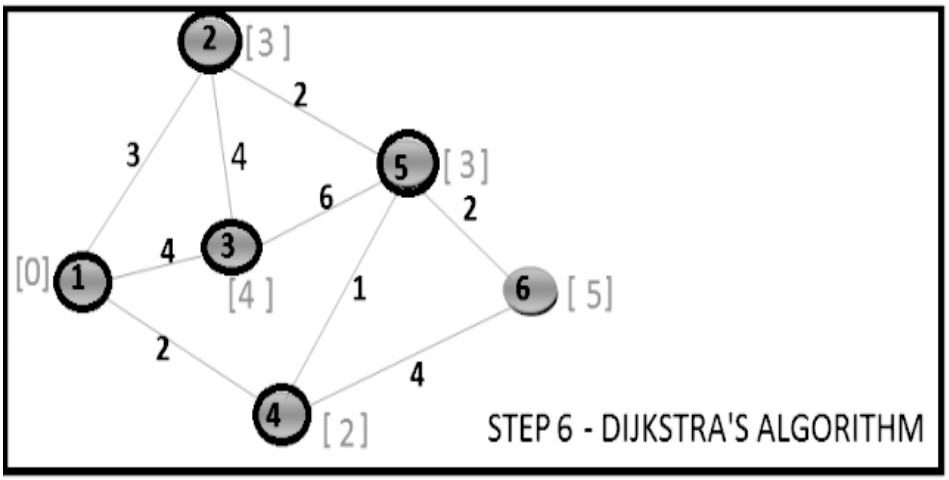
Choose the unvisited vertex with minimum cost (vertex 5) and consider all its unvisited neighbors (Vertex 3 and Vertex 6) and calculate the minimum cost for both of them. Now, the current cost of Vertex 3 is [4] and the sum of (cost of Vertex 5 + cost of edge (5,3)) is 3 + 6 = [9]. Minimum of 4, 9

is 4. Hence the cost of vertex 3 won't change. Now, the current cost of Vertex 6 is [6] and the sum of (cost of Vertex 5 + cost of edge (3,6)) is 3 + 2 = [5]. Minimum of 6, 5 is 45. Hence the cost of vertex 6 changes to 5. The state is as follows:



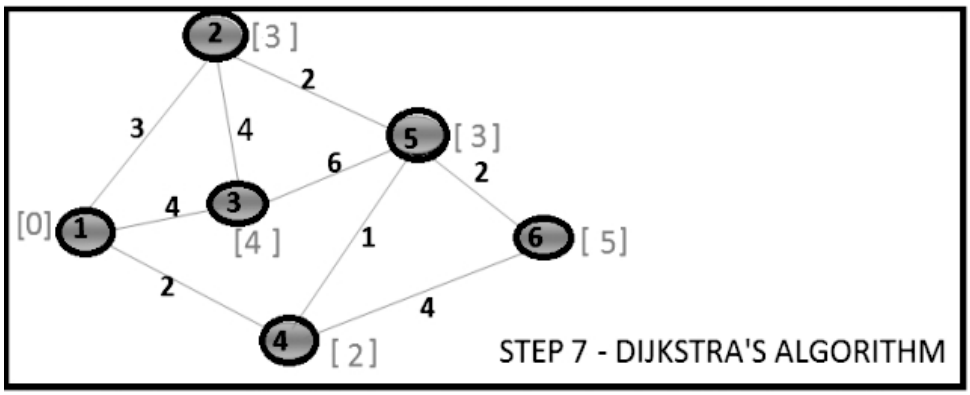
Step 6:

Choose the unvisited vertex with minimum cost (vertex 3) and consider all its unvisited neighbors (none). So mark it visited. The state is as follows:



Step 7:

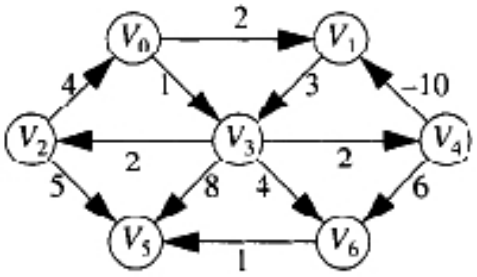
Choose the unvisited vertex with minimum cost (vertex 6) and consider all its unvisited neighbors (none). So mark it visited. The state is as follows:



Now there is no unvisited vertex left and the execution ends. At the end we know the shortest paths for all the vertices from the source vertex 1. Even if we know the shortest path length, we do not know the exact list of vertices which contributes to the shortest path until we maintain them separately or the data structure supports it.

9.10 BELLMAN-FORD ALGORITHM

Dijkstra's algorithm requires that edge costs be non-negative. This requirement is reasonable for most graph applications, but sometimes it is too restrictive. Hence, here we consider the most general case i.e. the negative-weighted, shortest-path algorithm. Therefore our objective is to find the shortest path (measured by total cost) from a designated vertex *S* to every vertex, edge costs may be negative. Let us consider a graph as shown below:



In this graph the path from V3 to v4 has a cost of 2. However, a shorter path exists by following the loop V_3, V_4, V_1, V_3, V_4. This has a cost of -3. This path is still not the shortest because we could stay in the loop arbitrarily long. Thus, the shortest path between these two points is undefined. This problem is not restricted to nodes in the cycle. The shortest path from V2 to V5 is also undefined because there is a way to get into and out of the loop. This loop is called a **negative-cost cycle,** which when present in a graph makes most, if not all, the shortest paths undefined. Negative-cost edges by themselves are not necessarily bad; it is the cycles that are. The general algorithm either finds the shortest paths or

reports the existence of a negative-cost cycle is known as the ***Bellman – Ford algorithm.***

Now we formally define the ***Bellman-Ford algorithm***, this algorithm accept the edge-weighted directed graph ***G = (V, E, w)*** and source ***s Є V*** as the input and return a shortest path tree.

Algorithm Bellman-Ford algorithm

1. *Initialize d and p.*
2. *For I = 1,….., |V|-1 do*
3. *For each e = (u, v) Є E do*

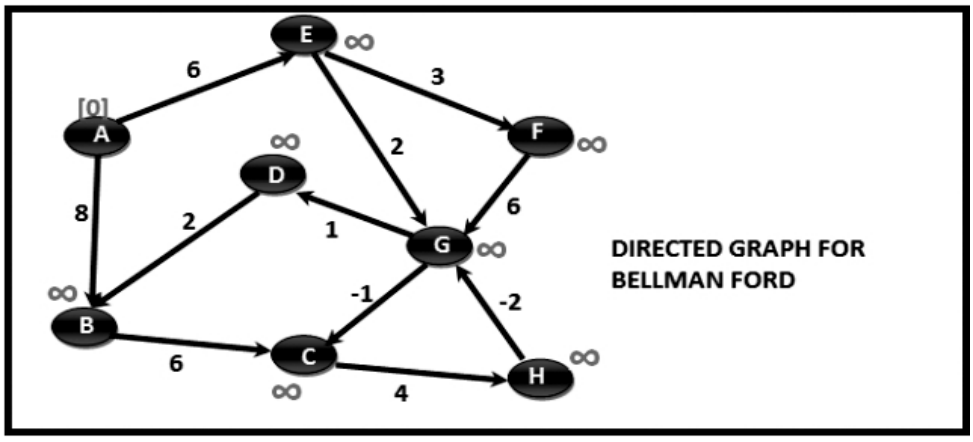
   if d [v] > d[u] + w((u, v)) then

   d [v] = d [u] + w ((u, v))

   p [v] = u;
4. *For each (u, v) Є E do*
5. *If d [v] > d [u] + w ((u, v)) then*
6. *Error "negative cycle"*
7. *Exit.*

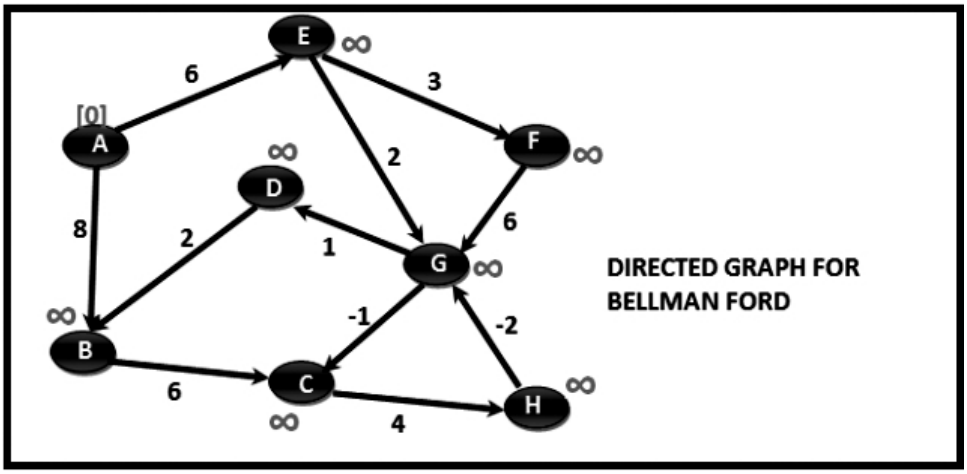
Example

The following implementation is showing the stages of the ***Bellman-Ford algorithm*** as follows for the given graph:



Step 1 :

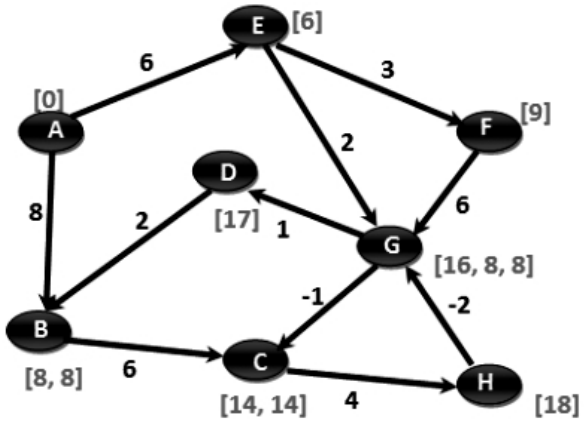
Considering A as the source, assign it the cost zero. Add all the vertices (A, B, C, D, E, F, G, H) to a list. For all vertices except A assign a cost infinity. Also, it is advisable to maintain a list of edges handy. Here is the graph to start with:



Step 2 :

Take one vertex at a time say A, and relax all the edges in the graph. Point worth noticing that we can only relax the edges which are out going from the vertex A. Rest of the edges will not make much of a difference. So, the following are the sub steps for step 2.

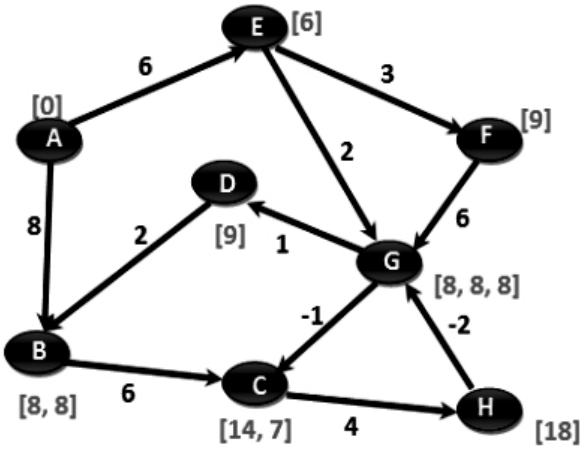
Relax (A, E) : cost of E = MIN(current cost of E[∞] , cost of A[0] + $W_{\{A,E\}}$[6]).Cost(E) becomes 6.
Relax (A, B) : cost of B = MIN(current cost of B[∞] , cost of A[0] + $W_{\{A,B\}}$[8]).Cost(B) becomes 8.
Relax (B, C) : cost of C = MIN(current cost of C[∞] , cost of B[8] + $W_{\{B,C\}}$[6]).Cost(C) becomes 14.
Relax (C, H) : cost of H = MIN(current cost of H[∞] , cost of C[14] + $W_{\{C,H\}}$[4]).Cost(H) becomes 18.
Relax (H, G) : cost of G = MIN(current cost of G[∞] , cost of H[18] + $W_{\{H,G\}}$[-2]).Cost(G) becomes 16.
Relax (G, C) : cost of C = MIN(current cost of C[14] , cost of G[16] + $W_{\{G,C\}}$[-1]).Cost(C) remains 14.
Relax (G, D) : cost of D = MIN(current cost of D[∞] , cost of G[16] + $W_{\{G,D\}}$[1]).Cost(D) becomes 17.
Relax (D, B) : cost of B = MIN(current cost of B[8] , cost of D[17] + $W_{\{D,B\}}$[2]).Cost(B) remains 8 .
Relax (E, F) : cost of F = MIN(current cost of F[∞] , cost of E[6] + $W_{\{E,F\}}$[3]).Cost(F) becomes 9.
Relax (E, G): cost of G = MIN(current cost of G[16] , cost of E[6] + $W_{\{E,G\}}$[2]).Cost(G) becomes 8.
Relax (F, G) : cost of G = MIN(current cost of G[8] , cost of F[9] + $W_{\{F,G\}}$[6]).Cost(G) remains 8.



Step 3:

Start from any one vertex, say A again and relax all the edges as below:

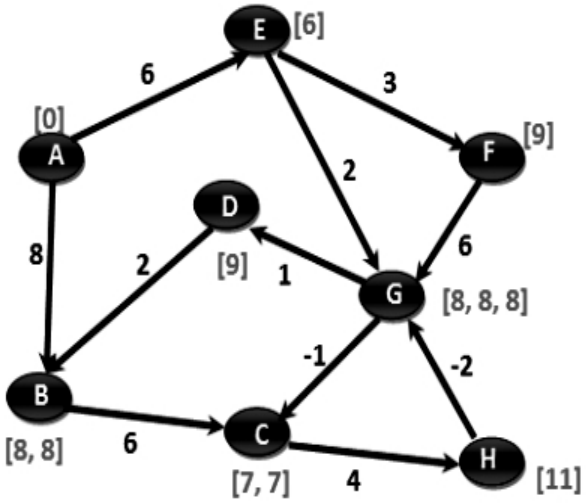
Relax (A, E) : cost of E = MIN(current cost of E[6] , cost of A[0] + $W_{\{A,E\}}$[6]).Cost(E) remains 6.
Relax (A, B) : cost of B = MIN(current cost of B[8] , cost of A[0] + $W_{\{A,B\}}$[8]).Cost(B) remains 8.
Relax (B, C) : cost of C = MIN(current cost of C[14] , cost of B[8] + $W_{\{B,C\}}$[6]).Cost(C) remains 14.
Relax (C, H) : cost of H = MIN(current cost of H[18] , cost of C[14] + $W_{\{C,H\}}$[4]).Cost(H) remains 18.
Relax (H, G) : cost of G = MIN(current cost of G[8] , cost of H[18] + $W_{\{H,G\}}$[-2]).Cost(G) remains 8.
Relax (G, C) : cost of C = MIN(current cost of C[14] , cost of G[8] + $W_{\{G,C\}}$[-1]).Cost(C) becomes 7.
Relax (G, D) : cost of D = MIN(current cost of D[17] , cost of G[8] + $W_{\{G,D\}}$[1]).Cost(D) becomes 9.
Relax (D, B) : cost of B = MIN(current cost of B[8] , cost of D[9] + $W_{\{D,B\}}$[2]).Cost(B) remains 8.
Relax (E, F) : cost of F = MIN(current cost of F[9] , cost of E[6] + $W_{\{E,F\}}$[3]).Cost(F) remains 9.
Relax (E, G) : cost of G = MIN(current cost of G[8] , cost of E[6] + $W_{\{E,G\}}$[2]).Cost(G) remains 8.
Relax (F, G) : cost of G = MIN(current cost of G[8] , cost of F[9] + $W_{\{F,G\}}$[6]).Cost(G) remains 8.



Step 4:

Start from any one vertex, say A again and relax all the edges as below:

Relax (A, E) : cost of E = MIN(current cost of E[6] , cost of A[0] + $W_{\{A,E\}}$[6]).Cost(E) remains 6.
Relax (A, B) : cost of B = MIN(current cost of B[8] , cost of A[0] + $W_{\{A,B\}}$[8]).Cost(B) remains 8.
Relax (B, C) : cost of C = MIN(current cost of C[7] , cost of B[8] + $W_{\{B,C\}}$[6]).Cost(C) becomes 7.
Relax (C, H) : cost of H = MIN(current cost of H[18] , cost of C[7] + $W_{\{C,H\}}$[4]).Cost(H) becomes 11.
Relax (H, G) : cost of G = MIN(current cost of G[8] , cost of H[11] + $W_{\{H,G\}}$[-2]).Cost(G) remains 8.
Relax (G, C) : cost of C = MIN(current cost of C[7] , cost of G[8] + $W_{\{G,C\}}$[-1]).Cost(C) remains 7.
Relax (G, D) : cost of D = MIN(current cost of D[9] , cost of G[8] + $W_{\{G,D\}}$[1]).Cost(D) remains 9.
Relax (D, B) : cost of B = MIN(current cost of B[8] , cost of D[9] + $W_{\{D,B\}}$[2]).Cost(B) remains 8.
Relax (E, F) : cost of F = MIN(current cost of F[9] , cost of E[6] + $W_{\{E,F\}}$[3]).Cost(F) remains 9.
Relax (E, G) : cost of G = MIN(current cost of G[8] , cost of E[6] + $W_{\{E,G\}}$[2]).Cost(G) remains 8.
Relax (F, G) : cost of G = MIN(current cost of G[8] , cost of F[9] + $W_{\{F,G\}}$[6]).Cost(G) remains 8.



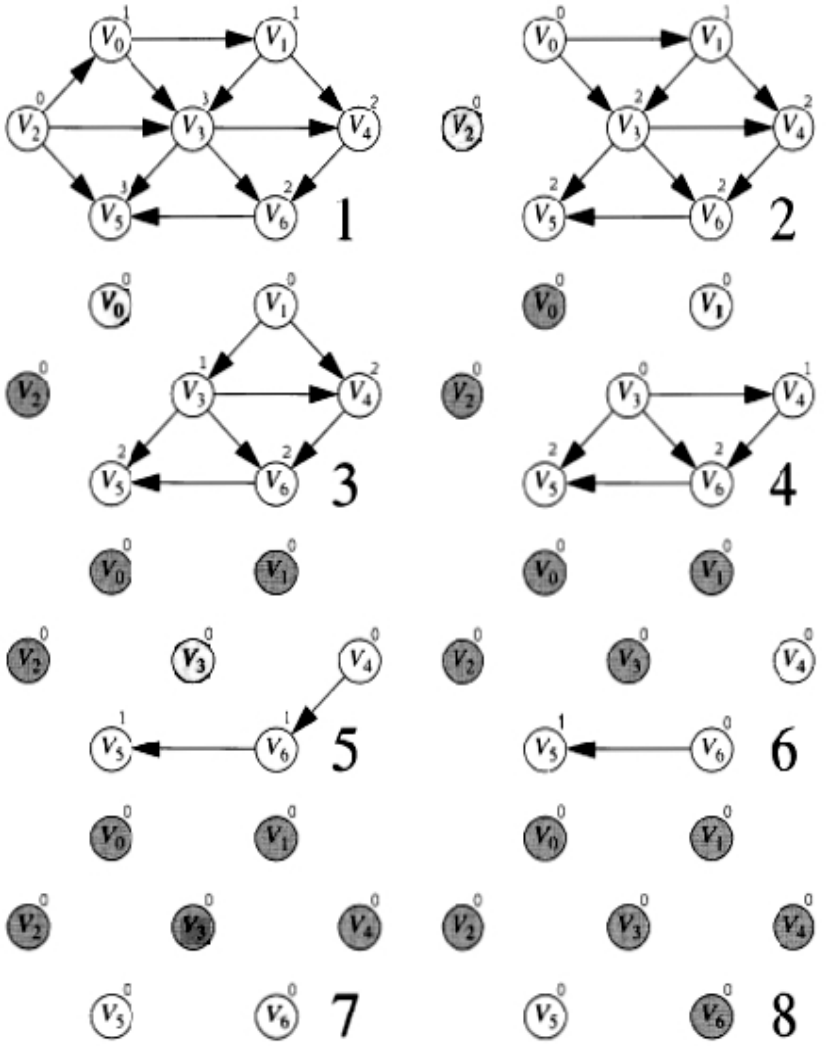
9.11 TOPOLOGICAL SORT

A Topological sort orders vertices in a directed acyclic graph such that if there is a path from *u* to *v*, then *v* appears after *u* in the ordering. For instance, a graph is typically used to represent the prerequisite requirement for courses at universities. An edge *(v, w)* indicates that course *v* must be completed before course *w* may be attempted. A topological order of the courses is any sequence that does not violate the prerequisite requirements. Thus, a Topological sort orders vertices in a directed acyclic graph such that if there is a path from u to v, then v appears after u in the ordering. A graph that has a cycle cannot have a topological order.

In this method we first find any vertex *v* that has no incoming edges. Then we print the vertex and logically move it, along with its edges, from the graph. Finally, we apply the same strategy to the rest of the graph. More formally, we say that the in–degree of vertex *v* is the number of incoming edges *(u, v).* We compute the in-degree of all vertices in the graph.

Example

Let us consider the following acyclic graph and represent the stages of topological sorting in this graph.



We apply the topological sorting method on the a cyclic graph by computing the in-degree for each vertex. Vertex V_2 has in-degree 0, so it is first in the topological order. If there were several vertices of in-degree 0, we could choose any one of them. When V_2 and its edges are removed from the graph, the in-degrees of V_0, V_3 and V_4 are all decremented by 1. Now V_0 has in-degree 0, so it is next in the topological order, and V_1 and V_3 have their in-degrees lowered. The algorithm continues, and the remaining vertices are examined in the order V_1, V_3, V_4, V_6 and V_5.

3.12 SUMMARY

In this unit we have discussed about the graph and its related terminology. The graph provides an excellent way to model the essential features of many applications, thereby facilitating specification of the underlying problems and formulation of algorithms for their solutions are discussed. The following contents were discussed:

- A Graph is a nonlinear data structure that is used to represent a relational data
- Each node of the graph is called as a ***vertex*** and link or line drawn between them is called and ***edge***
- The implementation of graph and its representations way were discussed. It has been seen that the graph is implemented in many ways by the use of different kinds of data structures.
- In many applications, edges are to be assigned costs or weights. Such graphs are known as weighted graphs.
- The directed graph and acyclic graphs were also described.
- There are various ways of representing a graph.
- Adjacency matrix of a graph is a matrix representation.
- Adjacency list of a graph is a linked list representation.
- The reach matrix and path matrix can also be constructed from the adjacency matrix representation of the graph.
- The method of obtaining the path matrix of a simple diagraph can easily be computed by using the *WARSHALL* algorithm
- There are two popular techniques for graph traversal i.e. breadth first search and Depth first search.
- A spanning tree of a graph is an undirected tree consisting of only those edges necessary to connect all the nodes in the original graph
- The two famous algorithms to complete a minimal spanning tree of a weighted graph are Kruskal's and prim's algorithm.
- The shortest path determination is important for any directed and weighted graph.
- For positive – weighted graphs the Dijkstr'a algorithm is used but for the negative weighted graphs the problem becomes more difficult.
- The general algorithm either finds the shortest paths or reports the existence of a negative-cost cycle is known as the ***Bellman – Ford algorithm.***
- Finally, for acyclic graphs, the running time reverts to linear time the aid of topological sort.

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SELF EVALUATION

1. A vertex with degree one is a graph is called
   a. A leaf
   b. Pendant Vertex
   c. Adjacency list
   d. None of the above
2. In an adjacency matrix parallel edges are given by:
   a. Similar Columns
   b. Similar rows
   c. Not represent-able
   d. None of the above
3. Which of the following representation of graph is more adequate?
   a. Stack representation of graphs.
   b. Adjacency matrix representation of graphs
   c. Linked representation of graphs.
   d. None of the above.
4. Prim's algorithm is a method available for finding out the minimum cost of a spanning tree.
   a. O(n*n)
   b. O(n log n)
   c. O (n)
   d. O (1)
5. What is a Path matrix?
6. Kruskal's algorithm for building minimal cost spanning tree of a graph considers edges for inclusion in the tree in the …… order of the cost.
7. Write Prim's algorithm for finding minimal spanning tree of any graph.
8. By considering the complete graph with n vertices, show that the number of spanning trees is at least $2^{n-1} - 1$
9. Find the shortest un-weighted path from V_3 to all others in the graph show in figure (A) below.

10. Find the shortest weighted path from V_2 to all others in the graph shown in figure (B)

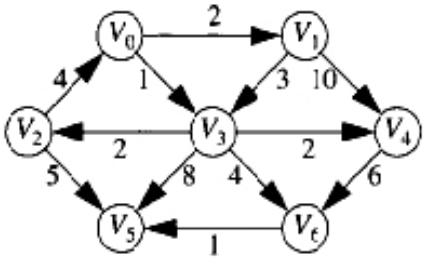

Figure (A)

11. Write depth first search algorithm for the traversal of any graph. Write a 'C' program for the same.

12. Define spanning tree and minimal spanning tree.

Uttar Pradesh Rajarshi Tandon
Open University

Postgraduate Diploma in Computer Application

PGDCA-107

Data Structure

BLOCK

4

Searching and Sorting, Hashing and File Organization

Curriculum Design Committee

Dr.P.P.Dubey — **Coordinator**
Director, School of Agri. Sciences,
UPRTOU, Prayagraj

Prof. U. N. Tiwari — **Member**
Dept. of Computer Science and Engg.,
Indian Inst. Of Information Science and Tech.,
Prayagraj

Prof. R.S. Yadav, — **Member**
Dept. of Computer Science and Engg.,
MNNIT, Allahabad, Prayagraj

Prof. P. K. Mishra — **Member**
Dept. of Computer Science,
Baranas Hindu University, Varanasi

Mr. Prateek Kesrwani — **Member Secretary**
Academic Consultant-Computer Science
School of Science, UPRTOU, Prayagraj

Course Design Committee

Prof. U. N. Tiwari — **Member**
Dept. of Computer Science and Engg.,
Indian Inst. Of Information Science and Tech.,
Prayagraj

Prof. R.S. Yadav, — **Member**
Dept. of Computer Science and Engg.,
MNNIT, Prayagraj

Prof. P. K. Mishra — **Member**
Dept. of Computer Science,
Baranas Hindu University, Varanasi

Faculty Members, School of Sciences

Dr. Ashutosh Gupta, Director, School of Science, UPRTOU, Prayagraj

Dr. Shruti, A sst. P rof., (Statistics), School of S cience, UPRTOU, Prayagraj

Ms. M arisha Asst. P rof., (Computer S cience), School of S cience, UPRTOU, Prayagraj

Mr. Manoj K Balwant Asst. Prof., (Computer Science), School of Science, UPRTOU, Prayagraj

Dr. Dinesh K Gupta Academic Consultant (Chemistry), Scool of Science, UPRTOU, Prayagraj

Course Preparation Committee

Prof. Manu Pratap Singh, **Author**
Dept. of Computer Science
Dr. B. R. Ambedkar University, Agra-282002

Dr. Ashutosh Gupta **Editor**
Director, School of Sciences,
UPRTOU, Prayagraj

Prof. U. N. Tiwari **Member**
Dept. of Computer Science and Engg.,
Indian Inst. Of Information Science and Tech.,
Prayagraj

Prof. R.S. Yadav **Member**
Dept. of Computer Science and Engg.,
MNNIT, Allahabad, Prayagraj

Prof. P. K. Mishra **Member**
Dept. of Computer Science
Baranas Hindu University, Varanasi

Dr. Dinesh K Gupta, **SLM Coordinator**
Academic Consultant- Chemistry School of Science, UPRTOU, Prayagraj

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RIL-102

BLOCK INTRODUCTION

This block will cover the various searching & sorting techniques, hashing techniques and file organization in the system. We will concentrate on some techniques to search a particular data or piece of information from a large amount of data in Unit 10. There are basically two types of searching techniques, linear or sequential search and binary search. We will also discuss various sorting algorithms like Selection sort, Bubble sort, Insertion sort, Heap sort, Quick Sort, Merge sort, Shell sort and Radix sort. Enough number of examples is discussed to show the operations in searching & sorting.

Hashing technique is also discussed in Unit 11. Hashing is the process of mapping large amount of data item to smaller table with the help of hashing function. Hashing is also known as Hashing Algorithm. It is a technique to convert a range of key values into a range of indexes of an array. It is used to facilitate the next level searching method when compared with the linear or binary

In common terminology, a file is a block of important data which is available to any computer software and is usually stored on any storage device. Storing a file on any storage medium like pen drive, hard disk or floppy disk ensures the availability of the file in future. Now days all file are stored in computers to reduce paper work and easy availability in any office, bank or library. The file organization is also covered in Unit 12 with examples.

This block will help you to realize the concept of searching, sorting, hashing and file organization in detail with suitable examples and with the help of codes.

UNIT-10 SEARCHING AND SORTING

Structure

10.0 INTRODUCTION

Searching is the process of looking for something: Finding one piece of data that has been stored within a whole group of data. However, if the value is not present in the array, the searching process displays an appropriate message and in this case searching is said to be unsuccessful. It is often the most time-consuming part of many computer programs. There are a variety of methods, or algorithms, used to search for a data item, depending on how much data there is to look through, what kind of data it is, what type of structure the data is stored in, and even where the data is stored - inside computer memory or on some external medium.

Sorting means arranging the elements of an array in smallest to largest (ascending) or largest to smallest (descending) order. A sorting algorithm is defined as an algorithm that puts the elements of a list in a certain order. Sorting algorithms are widely used to optimize the use of other algorithms like search and merge algorithms which require sorted lists to work correctly. There are two types of sorting:

- Internal sorting which deals with sorting the data stored in the computer's main memory (RAM)

- External sorting which deals with sorting the data stored in files (Secondary Storage)

In this unit, we will concentrate on some techniques to search a particular data or piece of information from a large amount of data. There are basically two types of searching techniques, Linear or Sequential Search and Binary Search. We will also discuss various sorting algorithms like Selection sort, Bubble sort, Insertion sort, Heap sort, Quick Sort, Merge sort, Shell sort and Radix sort.

10.1 OBJECTIVES

After going through this unit, you should be able to:

- Understand the concept of searching and its types.
- Implementation of sequential and binary with example.
- Understand the concept of sorting and its types.
- Understand the concept of Selection sort with example.
- Understand the concept of Bubble sort with example.
- Understand the concept of Insertion sort with example.
- Understand the concept of Heap sort with example.
- Understand the concept of Quick sort with example.
- Understand the concept of Merge sort with example.
- Understand the concept of Shell sort with example.
- Understand the concept of Radix sort with example.

10.2 SEQUENTIAL SEARCH

Sequential search is very simple to implement for searching an item in a collection. It is also known as ***Linear search***. It is not the most efficient way to search for an item in a collection. It works by comparing the value to be searched with every element of the array one by one in a sequential manner until an item is found. Moreover, if the array elements are arranged in random order, it is the only reasonable way to search an item. Thus, for many situations, Sequential search is a valid approach.

For example, if an array A [ ] is initialized as under,

int A[ ] = {25, 18, 22, 67, 33, 54, 69, 10, 87,78};

and the value to be searched is 67. So we have to search whether the value VAL=67 is present in the array or not. If yes, then it returns the position of its occurrence. Here, POS = 3 (index starting from 0).

Algorithm for Linear Search

```
Linear_Search (A, N, VAL)

Step 1: [initialize] SET pos = -1

Step 2: [initialize] SET idx = 1

Step 3:

        Repeat Step 4 WHILE idx<=N

Step 4:

                IF A[idx]= VAL

        SET pos=idx

        PRINT pos

        Go to Step 6

                [END OF IF]

        [END OF LOOP]

Step 6: EXIT

SET idx=idx+1



Step 5: IF pos =–1

        PRINT “VALUE IS NOT PRESENT IN THE ARRAY”

[END OF IF]
```

In Steps 1 and 2 of the algorithm, we shall initialize the value of ***pos*** and ***idx***. In next Step 3, a while loop is starting which will execute till ***idx*** is less than N (total number of elements). In Step 4, condition is checked to see if the ***VA***L is found at A [***idx***]. If VAL is found, then the position of the array element is printed, else the value of ***idx*** is incremented to match the next element with ***VAL***. However, if all the array elements have been

compared with ***VAL*** and no match is found, then it means that ***VAL*** is not present in the array.

Complexity of Linear Search Algorithm

Linear search executes in O(n) time where n is the number of items in the array. Definitely, the best case of linear search is when VAL is available at first location or zero index of the array. In this case, only one comparison will be made. In the same way, the worst case will happen when VAL is present in the last location of the array or it is not present in the array. In these both the cases, all N comparisons will have to be made.

Example: Write a program in 'C' language to search an element in an array using the sequential search technique

```
# include<stdio.h>

# include<conio.h>

int main()

{       intarr[20],n,i,item;

        clrscr();

        printf("How many elements you want to enter in the array : ");

        scanf("%d",&n);

        for(i=0; i<n;i++)

        {       printf("Enter element %d : ",i+1);

                scanf("%d", &arr[i]);

        }

        printf("Enter the element to be searched : ");

        scanf("%d",&item);
```

```
        for(i=0;i <n;i++)
        {       if(item == arr[i])
                {       printf("%d found at position %d\n",item,i+1);
                        break;
                }
        }/*End of for*/
        if(i == n)
                printf("Item %d not found in array\n",item);
return 0;
}
```

Efficiency of Linear Search:

In sequential search, we have seen that the number of comparisons depends upon the size of the array. If the required item is at the first place, then number of comparison is only '1'. If required item is at last position, 'n' comparisons have to make.

If item is at any position in the array, then, a successful search will take (n+1)/2 comparisons and an unsuccessful search will take 'n' comparisons. In any case, the order of the above algorithm is O(n).

10.3 BINARY SEARCH

Binary search is a searching algorithm that works efficiently with a sorted list of items. The sequential search situation will be in worst case if the element is at the end of the list. For eliminating this problem, we have one efficient searching technique called binary search. The condition for binary search is that all the data should be in sorted array. We compare the element with middle element of the array. If it is less than the middle element then we search it in the left portion of the array and if it is greater than the middle element then search will be in the right portion of the array. Now we will take that portion (either left or right) only for search and compare with middle element of that portion. This process will be repeated until we find required element or middle element has no left or right element.

For understanding this searching method, let us take 3 variables ***Beg***, ***End*** and ***Mid***. The Beg variable will indicate to first item's index of array (zero index of array), End variable will indicate to the last index of array. Mid will take care of average of both Beg and End as under:

$$Mid = (Beg + End) / 2$$

To understand the concept of binary search, let us take an array of 10 elements which is as under in ascending order.

Array [10]	0	1	2	3	4	5	6	7	8	9
Values	**12**	**34**	**39**	**45**	**67**	**73**	**85**	**93**	**102**	**110**

Now we are searching the item 85 in the above array.

12	**34**	**39**	**45**	**67**	**73**	**85**	**93**	**102**	**110**

Step 1: Beg = 0 End = 9 Mid = (Beg + End) / 2 = 4

0	1	2	3	**4**	5	6	7	8	9
12	**34**	**39**	**45**	**67**	**73**	**85**	**93**	**102**	**110**

Now at index 4, middle item is 67 which is smaller than required item 85. So we will search in right side from middle point. Now, Beg = Mid + 1 = 5, and End will be the same.

Step 2: Beg = 5 End = 9 Mid = (Beg + End) / 2 = 7

0	1	2	3	4	5	6	**7**	8	9
12	**34**	**39**	**45**	**67**	**73**	**85**	**93**	**102**	**110**

Now at index 7, middle item is 93 which is greater than required item 85. So we will search in left side from middle point. Now, End = Mid - 1 = 6, and Beg will be the same.

Step 3: Beg = 5 End = 6 Mid = (Beg + End) / 2 = 5

0	1	2	3	4	5	6	**7**	8	9
12	**34**	**39**	**45**	**67**	**73**	**85**	**93**	**102**	**110**

Now at index 5, middle item is 73 which is less than required item 85. So we will search in right side from middle point. Now, Beg = Mid+ 1 = 6, and End will be the same.

Step 4: Beg = 6 End = 6 Mid = (Beg + End) / 2 = 6

0	1	2	3	4	5	**6**	7	8	9
12	**34**	**39**	**45**	**67**	**73**	**85**	93	**102**	**110**

Now at index 6, middle item is 85 which is equal to required item 85.

Let see the algorithm of binary search.

```
BINARY_SEARCH(A, LB, UB, VAL)

Step 1: [INITIALIZE]

        SET BEG= LB

        END= UB,

        POS= -1

Step 2: Repeat Step 3 and 4 while BEG <= END

Step 3: SET MID= (BEG+ END)/2

Step 4: IF A[MID]= VAL
```

```
            SET POS= MID

            PRINT POS

            Go to Step 6

      ELSE IF A[MID]> VAL

            SET END= MID-1

      ELSE

            SET BEG= MID+1

      [END OF IF]

      [END OF LOOP]
```

Step 5: IF POS=-1

PRINT “SEARCHED ITEM NOT FOUND IN THE ARRAY”

[END OF IF]

Step 6: EXIT

Example:

Write a program in ‘C’ language to search an element in an array using the binary search technique

```
#include <stdio.h>

# include<conio.h>

int main()

{     intarr[20],start,end,middle,n,i,item;

      printf("How many elements you want to enter in the array : ");

      scanf("%d",&n);
```

```
        for(i=0; i< n; i++)
        {       printf("Enter element %d : ",i+1);
                scanf("%d",&arr[i]);
        }
        printf("Enter the element to be searched : ");
        scanf("%d",&item);
        start=0;
        end=n-1;
        middle=(start+end)/2;
        while(item != arr[middle] && start <= end)
        {       if(item >arr[middle])
                        start=middle+1;
                else
                        end=middle-1;
                middle=(start+end)/2;
        }
        if(item==arr[middle])
                printf("%d found at position %d\n",item,middle+1);
        if(start>end)
                printf("%d not found in array\n",item);
 return 0;
}
```

Efficiency of Binary Search:

In the binary search algorithm, we see that with each comparison, the size of the array where search has to be made is reduced to half. Thus, the maximum number of key comparisons are approximately $\log_2 n$. So, the order of binary search is $O(\log_2 n)$.

Comparison of Linear Search and Binary Search:

Binary search is faster than linear search. Here are some comparisons. We are taking average case for both the search methods to be compare.

Array size	Sequential search	Binary search
8	4	4
128	64	8
256	128	9
1000	500	11
100000	50000	18

10.4 SORTING

Suppose you have to retrieve any information which is stored in some predefined order than it is very easy task. If information is not in any order than you have to search that from beginning to till end. So, Sorting is a very important computer application activity. Sorting is a technique for rearranging the elements of a list in ascending or descending order.

There are lot of sorting algorithms available. But depend on the different environments, different sorting methods are used. Broadly Sorting can be classified in two types i.e. Internal and External Sorting.

Internal Sorting : This method uses only the primary memory during sorting process. All data items are held in main memory and no secondary memory is required this sorting process. If all the data that is to be sorted can be accommodated at a time in memory is called internal sorting.

There is a limitation for internal sorts; they can only process relatively small lists due to memory constraints. There are 3 types of internal sorts.

1. EXCHANGE SORT :- Ex:- Bubble Sort Algorithm, Quick sort algorithm

2. INSERTION SORT :- Ex:- Insertion sort algorithm, Shell Sort algorithm

3. SELECTION SORT :- Ex:- Selection sort algorithm, Heap Sort algorithm

External Sorting : Sorting large amount of data requires external or secondary memory. This process uses external memory such as HDD, to store the data which is not fit into the main memory. So, primary memory holds the currently being sorted data only. All external sorts are based on process of merging. Different parts of data are sorted separately and then merged together. Ex: Merge Sort

10.5 SELECTION SORT

Selection sort is the process to select the smallest element from the array and put it at first place (zero index) in the array. Suppose A is an array of size n stored in memory. The selection sort algorithm first selects the smallest element in the array A and place it in array at index 0; then it selects the next smallest element in the array A and place it in array at index 1 and so on. It simply continues this procedure until it places the biggest element in the last position of the array.

The array is passed through (n-1) times and the smallest element is placed in its respective position in the array as detailed below:

Pass 1: Find the location j of the smallest element in the array x [0], A [1],....A [n-1], and then interchange x[j] with A [0]. Then A [0] is sorted.

Pass 2: Leave the first element and find the location j of the smallest element in the sub-array A [1], A [2],....A[n-1], and then interchange A[1] with A[j]. Then A[0], A[1] are sorted.

Pass 3: Leave the first two elements and find the location j of the smallest element in the sub-array A [2], A [3],....A[n-1], and then interchange A [2] with A [j]. Then A [0], A [1], A[2] are sorted.

Pass (n-1): Find the location j of the smaller of the elements A[n-2] and A[n-1], and then interchange A[j] and A[n-2]. Then A[0], A[1], ... A[n-2] are sorted. Of course, during this pass A [n-1] will be the biggest element and so the entire array is sorted.

0	1	2	3	4	5	6	7	8	Remarks
65	70	75	80	50	60	55	85	**45**	find the smallest element
i								j	swap a[i] & a[j]
45	70	75	80	**50**	60	55	85	65	find the second smallest
	i			j					swap a[i] & a[j]
45	50	75	80	70	60	**55**	85	65	Find the third smallest
		i				j			swap a[i] & a[j]
45	50	55	80	70	**60**	75	85	65	Find the fourth smallest
			i		j				swap a[i] & a[j]
45	50	55	60	70	80	75	85	**65**	Find the fifth smallest
				i				j	swap a[i] & a[j]
45	50	55	60	65	80	75	85	**70**	Find the sixth smallest
					i			j	swap a[i] & a[j]
45	50	55	60	65	70	**75**	85	80	Find the seventh smallest
						i j			swap a[i] & a[j]
45	50	55	60	65	70	75	85	**80**	Find the eighth smallest
							i	j	swap a[i] & a[j]
45	50	55	60	65	70	75	80	85	The outer loop ends.

Complexity of Selection Sort

In general we prefer selection sort in case where the insertion sort or the bubble sort requires exclusive swapping. In spite of superiority of the selection sort over bubble sort and the insertion sort (there is significant decrease in run time), its efficiency is also $O(n^2)$ for n data items.

Programming Example of Selection Sort

```
/*Program of sorting using selection sort*/
#include <stdio.h>
#include <conio.h>
int main()
{       inti,j,k,n=9;
        intarr[9]={65,70,75,80,50,60,55,85,45};
        clrscr();
        printf("Given Unsorted list is :\n");
        for (i = 0; i< n; i++)
                printf("%3d ", arr[i]);
/*Selection sort*/
        for(i = 0; i< n - 1 ; i++)
        {       /*Find the index of smallest element*/
                smallest = i;
                for(k = i + 1; k < n ; k++)
                {       if(arr[smallest] >arr[k])
                        smallest = k ;
                }
                if( i != smallest )
                {       temp = arr [i];
                        arr[i] = arr[smallest];
                        arr[smallest] = temp ;
                }
                printf("\n After Pass %d elements are :  ",i+1);
                for (j = 0; j < n; j++)
                        printf("%d ", arr[j]);
        }
        printf("\n Sorted list is : \n");
        for (i = 0; i< n; i++)
                printf("%d ", arr[i]);
return 0;

}
```

10.6 BUBBLE SORT

The bubble sort is easy to understand and program. The basic idea of bubble sort is to pass through the file sequentially several times. In this sorting algorithm, multiple swapping take place in one pass. In each pass, we compare each element in the file with its successor i.e., A [i] with A [i+1] and interchange two element when they are not in proper order. We will illustrate this sorting technique by taking a specific example. Bubble sort is also called as exchange sort.

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Example: Let us take an array A [n] which is stored in memory as shown below:

Array [5]	0	1	2	3	4	5
Values	**34**	**45**	**24**	**12**	**67**	**56**

Suppose our objective is to store our array in ascending order. Then we follow steps through the array 5 times as below:

Pass 1: We will compare A[i] and A[i+1] for i = 0, 1, 2, 3, and 4, and swap A[i] with A[i+1], only if A[i] > A[i+1] otherwise move to next one. The process is shown below:

A[0]	A[1]	A[2]	A[3]	A[4]	A[5]
34	**45**	**23**	**12**	**67**	**56**
	23	45			
		12	45		
			45	67	
				56	67
34	23	12	45	56	67

In this first pass, the biggest number of the array 67 is moved to the right most position in the array.

Pass 2: Now once again, we repeat the same process. But be careful, this time we will not include A[5] in our comparisons as it is already moved to its proper location.

It means, we compare A[i] with A[i+1] for i =0, 1, 2, and 3 and interchange A[i] and A[i+1]

if A[i] > A[i+1]. The process is shown below:

A[0]	A[1]	A[2]	A[3]	A[4]	A[5]
34	**23**	**12**	**45**	**56**	**67**
23	34 12	34 34	45 45	56	
23	12	34	45	56	67

Pass 3: We repeat the same process, but this time we leave both A[4] and A [5] as these both are on correct position. By doing this, we move the third biggest number 45 to A [3].

A[0]	A[1]	A[2]	A[3]	A[4]	A[5]
23	**12**	**34**	**45**	**56**	**67**
12	23 23	34 34	45		
12	23	34	45	56	67

Pass 4: We repeat the process by leaving elements at A [3], A [4] and A[5]. By doing this, we move the fourth biggest number 34 to A[2].

A[0]	A[1]	A[2]	A[3]	A[4]	A[5]
12	**23**	**34**	**45**	**56**	**67**
12	23 23	33			
12	23	34	45	56	67

Pass 5: We repeat the process by leaving elements at A[2], A[3], A[4] and A[5]. By doing this, we move the fifth biggest number 23 to A[1]. At this time, we will have the smallest number 12 in A[0]. Thus, we see that we can sort the array of size 6 in 5 passes.

A[0]	A[1]	A[2]	A[3]	A[4]	A[5]
12	**23**	**34**	**45**	**56**	**67**
12	23				
12	23	34	45	56	67

For an array of size n, we required (n-1) passes using bubble sort.

Algorithm for bubble sort:

```
BUBBLE_SORT(A, N)
Step 1: Repeat Step2 For I=0 to N-1
Step 2: Repeat For J= 0 to N-I
Step 3: IF A[J]> A[J+1]
            SWAP A[J] and A[J+1]
            [END OF INNER LOOP]
            [END OF OUTER LOOP]
Step 4: EXIT
```

Complexity of Bubble Sort:

The complexity of any sorting algorithm depends on how many comparisons are there in it. In bubble sort, there are total N–1 passes. In the 1st pass, N–1 comparisons, then in 2nd Pass, there are N–2 comparisons and so on. Therefore complexity can be given as:

$f(n) = (n – 1) + (n – 2) + (n – 3) + ... + 3 + 2 + 1$

$f(n) = n (n – 1)/2$

$f(n) = n^2/2 + O(n) = O(n^2)$

Therefore, we can see that, it is in the form of arithmetic progression which is of $O(n^2)$. Therefore bubble sort is very inefficient when there are more elements to sorting.

Programming Example of Bubble Sort :

```
#include<stdio.h>
#include<conio.h>
int main()
{       intarr[5]={34,45,23,12,67,56};
        inti,j,k,tmp,n,xchanges;
        clrscr();
        printf("The given list is :\n");
        for (i = 0; i< n; i++)
                printf("%3d ", arr[i]);
        printf("\n");
        /* Bubble sort procedure*/
        for (i = 0; i< n-1 ; i++)
        {       xchanges=0;
                for (j = 0; j <n-1-i; j++)
                {  if (arr[j] >arr[j+1])
                        {       tmp = arr[j];
                                arr[j] = arr[j+1];
                                arr[j+1] = tmp;
                                xchanges++;
                        }
                }
        printf("After Pass %d elements are :  ",i+1);
        for (k = 0; k < n; k++)
                printf("%3d ", arr[k]);
        printf("\n");
    }
        printf("Sorted list is :\n");
        for (i = 0; i< n; i++)
                printf("%3d ", arr[i]);
        printf("\n");
retun 0;
}
```

10.7 INSERTION SORT

Insertion sort is a very simple sorting algorithm in which the sorted array is built by one element at a time. In insertion sort the element is inserted at an appropriate place similar to card insertion, as we see in playing card. Both the selection and bubble sorts exchange the elements to make all in order. But insertion sort does not exchange elements. Here the list is divided into two parts sorted and unsorted. In each pass, the first element of unsorted sub list is picked up and moved into the sorted sub list by inserting it in suitable position just like we arrange the playing cards.

Let us take an example to show how Insertion sort works to sort an array Arr of size 10 as below:

Arr[10]	0	1	2	3	4	5	6	7	8	9
Values	41	11	47	65	20	83	110	56	74	38

41	11	47	65	20	83	110	56	74	38	Now Arr[0] is first element and sorted.
11	41	47	65	20	83	110	56	74	38	Pass 1, first two items are sorted.
11	41	47	65	20	83	110	56	74	38	Pass 2, first three items are sorted.
11	41	47	65	20	83	110	56	74	38	Pass 3, first four items are sorted.
11	20	41	47	65	83	110	56	74	38	Pass 4, first five items are sorted.
11	20	41	47	65	83	110	56	74	38	Pass 5, first six items are sorted.
11	20	41	47	65	83	110	56	74	38	Pass 6, first seven items are sorted.
11	20	41	47	56	65	83	110	74	38	Pass 7, first eight items are sorted.
11	20	41	47	56	65	74	83	110	38	Pass 8, first nine items are sorted.
11	20	38	41	47	56	65	74	83	110	Pass 9, All 10 items are sorted.

In starting, Arr[0] is the only element which is sorted. In Pass 1, Arr[1] will be placed before or after Arr[0], so that the two item of array Arr will be sorted. In Pass 2, Arr[2] will be placed either before Arr[0], in between Arr [0] and Arr [1], or after Arr [1] and first three item will be sorted. In the same way, in Pass N–1, Arr [N–1] will be placed in its proper place to keep the array sorted.

Algorithm for insertion sort:

```
INSERTION-SORT (ARR, N)

Step 1: Repeat Steps 2 to 5 for K = 1 to N–1

Step 2: SET TEMP = ARR[K]

Step 3: SET J = K - 1

Step 4: Repeat while TEMP <= ARR[J]

            SET ARR[J+1]= ARR[J]

            SET J = J - 1

            [END OF INNER LOOP]

Step 5: SET ARR[J+1] = TEMP

      [END OF LOOP]

Step 6: EXIT
```

Complexity of insertion sort:

In insertion sort, we insert the element before or after and we start comparison from the first element. So First element has no previous element means no comparison. Second element needs 1 comparison, third element needs 2 comparison and so on last element needs n-1 comparison. It means total comparison will be

1+2+3+4+……. + n-2 + n-1.

which is again $O(n^2)$. It is the worst case when all elements are in reverse order. It is O(n) when elements are in sorted order .

Programming Example of Insertion Sort:

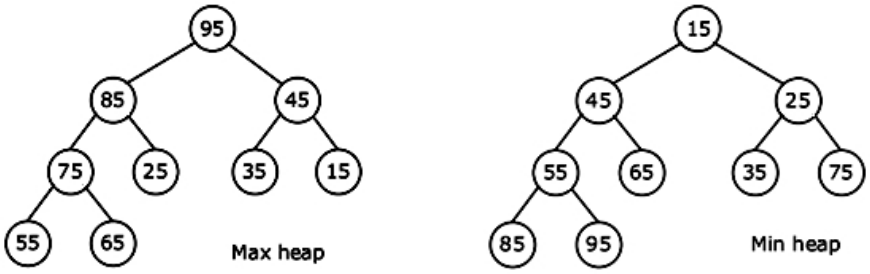
```
/* Program of sorting using insertion sort */
#include <stdio.h>
#include <conio.h>
int main()
{       inti,j,k,n=10;
        intarr[10]={41,11,47,65,20,83,110,56,74,38};
        printf("Given Unsorted list is :\n");
        for (i = 0; i< n; i++)
                printf("%3d ", arr[i]);
        /*Procedure for Insertion sort*/
        for(j=1;j<n;j++)
        {       k=arr[j];
                for(i=j-1;i>=0 && k<arr[i];i--)
                        arr[i+1]=arr[i];
                arr[i+1]=k;
                printf("\nPass %d,Element placed in proper place:%d",j,k);
                for (i = 0; i< n; i++)
                        printf("%d ", arr[i]);
                printf("\n");
        }
        printf("Sorted list is :\n");
        for (i = 0; i< n; i++)
                printf("%d ", arr[i]);
        printf("\n");
return 0;
}
```

10.8 HEAP SORT

Heap is a data structure, which permits one to insert elements into a set and also to find the largest element efficiently. A binary heap is a complete binary tree in which every node satisfies the heap property which states that:

If B is a child of A, then key(A) ≥ key(B)

This implies that elements at every node will be either greater than or equal to the element at its left and right child. Thus, the root node has the highest key value in the heap. Such a heap is commonly known as a **max-heap**.



Alternatively, elements at every node will be either less than or equal to the element at its left and right child. Thus, the root has the lowest key value. Such a heap is called a **min-heap**.

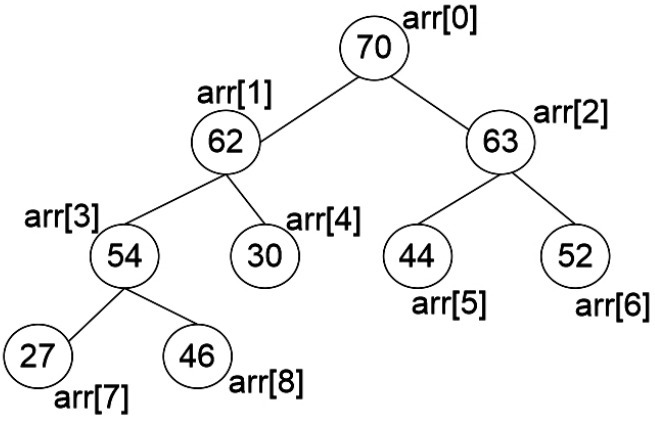
Heap sort is an improvement over the binary tree sort. It does not create nodes in case of Binary tree sort. Instead it builds a heap by adjusting the position of elements within the array itself.

The heap sort is sorting algorithm the efficiency of which is roughly equivalent to that of the quick sort. The three phases involved in sorting the elements using heap sort algorithm are as follows.

1. Construct a heap by adjusting the array elements.
2. Replace the root with the last node of heap tree.
3. Keep the last node (new root) at the proper position, means do not delete operation in heap tree but here deleted node is root.

The root element of a max heap is always the largest element. The sorting ends when all the root elements of each successive heap has been moved to the end of the array (i.e. when the tree is exhausted). The resulting array now contains a sorted list.

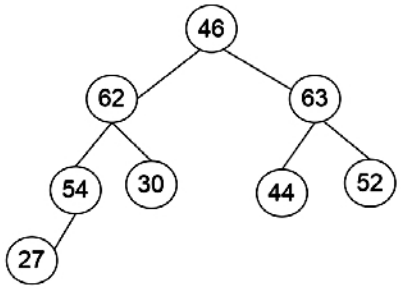
Let's take an example of heap sort using an array Arr of size 9.



Arr	0	1	2	3	4	5	6	7	8
Values	70	62	63	54	30	44	52	27	46

Figure A: Array with its equivalent Heap tree.

Step 1:

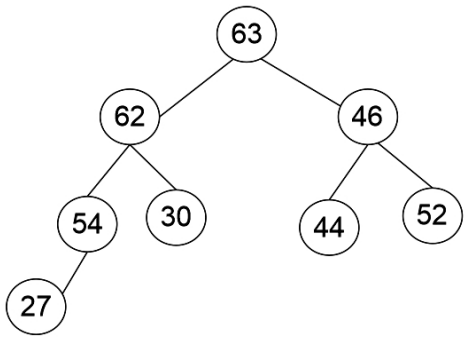


Arr	0	1	2	3	4	5	6	7	**8**
Values	46	62	63	54	30	44	52	27	**70**

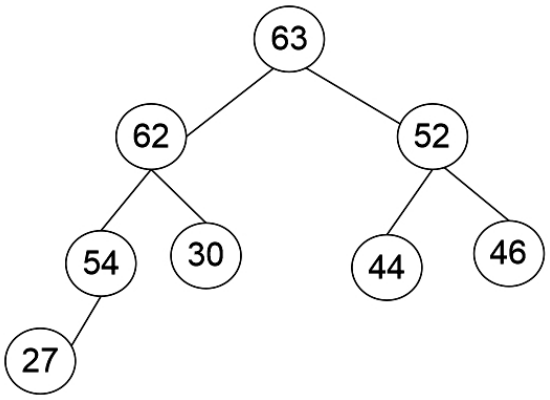
Figure B: Heap after eliminating root element 70.

In the same way, one by one the root element of the heap is eliminated, the following figure show the heap and array after each elimination.

In figure B root is at the position of the last node and the last node is at the position of root. Here left and right child of 46 is 62 and 63. Both are greater than 46, but right child 63 is greater than left child 62, hence replace it with the right child 63.



Here right child of 46 is 52, which is greater than 46, hence replace it with 52.

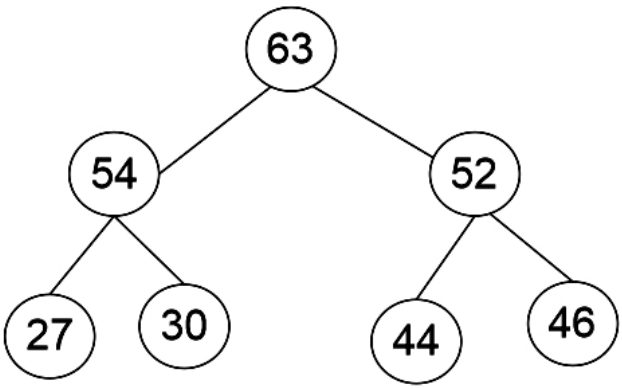


Now the elements of heap tree in array are as

Arr	0	1	2	3	4	5	6	7	**8**
Values	63	62	52	54	30	44	46	27	**70**

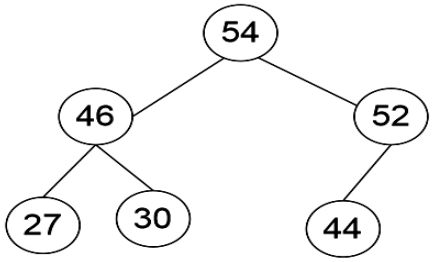
Now 27 is the last node. So replace it with root node 63 and do the same operation.

Step 2:



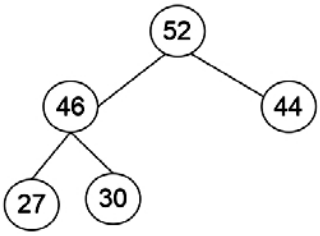
Arr	0	1	2	3	4	5	6	**7**	**8**
Values	63	54	52	27	30	44	46	**63**	**70**

Step 3:



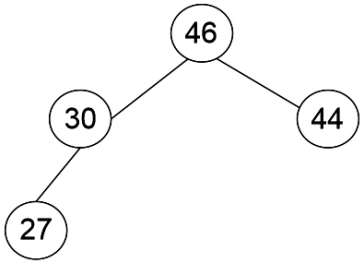
Arr	0	1	2	3	4	5	**6**	**7**	**8**
Values	54	46	52	27	30	44	**62**	**63**	**70**

Step 4:



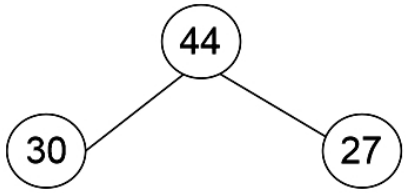
Arr	0	1	2	3	4	**5**	**6**	**7**	**8**
Values	52	46	44	27	30	**54**	**62**	**63**	**70**

Step 5:



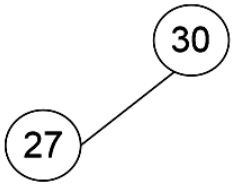
Arr	0	1	2	3	**4**	**5**	**6**	**7**	**8**
Values	46	30	44	27	**52**	**54**	**62**	**63**	**70**

Step 6:



Arr	0	1	2	**3**	**4**	**5**	**6**	**7**	**8**
Values	44	30	27	**46**	**52**	**54**	**62**	**63**	**70**

Step 7:



Arr	0	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Values	30	27	**44**	**46**	**52**	**54**	**62**	**63**	**70**

Step 8:

Arr	0	**1**	2	3	4	5	**6**	**7**	**8**
Values	27	**30**	**44**	**46**	**52**	**54**	**62**	**63**	**70**

Algorithm for heap sort:

```
        HEAPSORT(ARR, N)

Step 1: [Build Heap H]

        Repeat for I=0 to N-1

        CALL Insert_Heap(ARR, N, ARR[I])

        [END OF LOOP]



Step 2: (Repeatedly delete the root element)

        Repeat while N>

        CALL Delete_Heap(ARR, N, VAL)

        SET N = N + 1

        [END OF LOOP]

Step 3: END
```

Complexity of Heap Sort:

Let us consider the Timing Analysis of this heapsort. Since heap is an almost complete binary tree, the worst case analysis is easier than the average case. In order to sort a given array elements, we need to create a heap and then adjust it. This require number of comparisons in the worst case is **O (n log n).** The worst case behavior of heap sort is far superior to quick sort.

Programming Example:

```
/* Program of sorting through heapsort*/
# include <stdio.h>
intarr[20]={32,22,65,14,52,87,54,38,42,11};
int n=10;
main()
{       clrscr();
        printf("Entered list is :\n");
```

```
        display();
        create_heap();
        printf("Heap is :\n");
        display();
        heap_sort();
        printf("Sorted list is :\n");
        display();
}/*End of main()*/

display()
{       inti;
        for(i=0;i<n;i++)
                printf("%d  ",arr[i]);
        printf("\n");
}/*End of display()*/

create_heap()
{       inti;
        for(i=0;i<n;i++)
                insert(arr[i],i);
}/*End of create_heap()*/

insert(intnum,intloc)
{       int par;
        while(loc>0)
        {       par=(loc-1)/2;
                if(num<=arr[par])
                {       arr[loc]=num;
                        return;
                }
                arr[loc]=arr[par];
                loc=par;
        }/*End of while*/
        arr[0]=num;
}/*End of insert()*/


heap_sort()
{       int last;
        for(last=n-1; last>0; last--)
        del_root(last);
}

del_root(int last)
{       intleft,right,i,temp;
        i=0; /*Since every time we have to replace root with last*/
        /*Exchange last element with the root */
        temp=arr[i];
        arr[i]=arr[last];
        arr[last]=temp;
        left=2*i+1; /*left child of root*/
```

```
        right=2*i+2;/*right child of root*/
        while( right < last)
        {       if( arr[i]>=arr[left] &&arr[i]>=arr[right] )
                        return;
                if( arr[right]<=arr[left] )
                {       temp=arr[i];
                        arr[i]=arr[left];
                        arr[left]=temp;
                        i=left;
                }
                else
                {       temp=arr[i];
                        arr[i]=arr[right];
                        arr[right]=temp;
                        i=right;
                }
                left=2*i+1;
                right=2*i+2;
        }/*End of while*/
        if( left==last-1 &&arr[i]<arr[left] )/*right==last*/
        {       temp=arr[i];
                arr[i]=arr[left];
                arr[left]=temp;
        }
}/*End of del_root*/
```

10.9 QUICK SORT

The quick sort was invented by Prof. C. A. R. Hoare in the early 1960's. It was one of the first most efficient sorting algorithms. It is an example of algorithms that works on "divide and conquer" technique.

Quick sort is based on partition. It is also known as partition exchange sorting. The basic concept of quick sort process is pick one element from an array and rearranges the remaining elements around it. This element divides the main list into two sub lists. This chosen element is called pivot. Once pivot is chosen, then it shifts all the elements less than pivot to left of value pivot and all the elements equal or greater than pivot are shifted to the right side. When all the subsets have been partitioned and rearranged recursively, the original array is sorted.

How to choose pivot: So now the main task is to find the pivot element, which will divide the list into two partitions. Just assume first element as pivot and arrange the list accordingly into two halves.

The quick sort algorithm works as follows:

1. Select any random element **pivot** from the array elements.

2. Now rearrange the elements in such a way that all elements less than pivot appear in left of the pivot and all equal and greater elements should be in right side of the pivot. After such a partitioning, the pivot is placed in its final position.

3. Now recursively apply same procedure to both the sub list i.e. left to pivot and right to pivot. (One with sub-list of values smaller than that of the pivot element and the other having higher value elements.)

Example:

Let us take an array of 6 element and sort it using quick sort algorithm.

A[6]	0	1	2	3	4	5
Original Array	29	12	38	20	27	47

We choose the first element A[0] as the pivot ie **29**. Set Loc=0, Left=0 and Right =5.

29	12	38	20	27	47
Loc, Left					Right

Now look from right to left. Since A[Loc] < A[Right], decrease value of Right.

29	12	38	20	27	47
Loc, Left				Right	

Since A[Loc] > A[Right], swap both values and set Loc=Right.

27	12	38	20	**29**	47
Left				Loc, Right	

Now look from left to right. Since A[Loc] > A[Left], increment the value of left 2 times.

27	12	38	20	29	47
		Left		Loc, Right	

Since A[Loc] < A[Left], swap both values and set Loc=Left.

27	12	**29**	20	**38**	47
		Left, Loc		Right	

Now look from right to left. Since A[Loc] < A[Right], decrease the value of right.

27	12	**29**	20	**38**	47
		Left, Loc	Right		

Since A[Loc] > A[Right], swap the values and set Loc=Right

27	12	**20**	**29**	38	47
		Left	Loc, Right		

Now look from left to right. Since A[Loc] > A[Left], increase the value of left.

27	12	20	**29**	38	47
			Loc, Right, Left		

Now see the above last step in which Left = Loc. So terminates this process here. Now you can see that the pivot element 29 is at right place. All the smaller items are in left side and big items are in right side of pivot. Now same procedure will applied to both left and right list of pivot.

Algorithm of Quick sort :

The quick sort algorithm makes use of a function Quick to divide the array into two sub-arrays and fix the pivot at its suitable location.

PARTITION (ARR, BEG, END, LOC)

Step 1: [Initialize] Set Left= BEG, Right= END, LOC= BEG, Flag =0

Step 2: Repeat Step 3 To 6 While Flag =0

Step 3: Repeat While ARR[LOC] <= ARR[Right] And LOC != Right

Set Right= Right-1

[End Loop]

Step 4: If LOC= Right

Set Flag=1

Else If ARR[LOC]> ARR[Right]

Swap ARR[LOC] With ARR[Right]

Set LOC= Right

[End If]

Step 5: If Flag =0

Repeat While ARR[LOC] >= ARR[Left] And LOC != Left

Set Left= Left+1

[End Loop]

Step 6: If LOC= Left

Set Flag=1

```
        Else If ARR[LOC]< ARR[Left]
            Swap ARR[LOC] With ARR[Left]
            Set LOC= Left
        [End If]
        [End If]
Step 7: [End Loop]
Step 8: End
```

```
QUICK_SORT (ARR, BEG, END)
Step 1: IF (BEG< END)
            CALL PARTITION (ARR, BEG, END, LOC)
            CALL QUICKSORT(ARR, BEG, LOC-1)
            CALL QUICKSORT(ARR, LOC+1, END)
[END OF IF]
Step 2: END
```

Complexity of Quick sort :

The average runtime efficiency of the Quicksort is $O(n\log_2 n)$, which is the best that has been achieved for a large array of size n. in the worst case situation; when the array is already sorted, the efficiency of the Quicksort may drop down to $O(n^2)$ due to the continuous right-to-left scan all the way to the last left boundary.

Practically, the efficiency of quick sort depends on the element which ischosen as the pivot. Itsworst-case efficiency is given as $O(n^2)$. The worst case occurs when the array is already sorted (either in ascending or descending order) and the left-most element is chosen as the pivot.

Programming Example Quick sort:

```
#include <stdio.h>
#include <conio.h>
int partition(int a[], int beg, int end);
void quick_sort(int a[], int beg, int end);
void main()
{
inti, n=6;
intarr[6]={29,12,38,20,27,47};
 // clrscr();
quick_sort(arr, 0, n-1);
printf("\n The sorted array is: \n");
  for(i=0;i<n;i++)
printf(" %d\t", arr[i]);
getch();
}
int partition(int a[], int beg, int end)
{
int left, right, temp, loc, flag;
loc = left = beg;
  right = end;
  flag = 0;
  while(flag != 1)
  {
       while((a[loc] <= a[right]) && (loc!=right))
          right--;
       if(loc==right)
                flag =1;
        else if(a[loc]>a[right])
          {
               temp = a[loc];
               a[loc] = a[right];
               a[right] = temp;
loc = right;
             }
             if(flag!=1)
             {
                  while((a[loc] >= a[left]) && (loc!=left))
    left++;
if(loc==left)
   flag =1;
 else if(a[loc] <a[left])
    {
                                    temp = a[loc];
                                    a[loc] = a[left];
                                    a[left] = temp;
                                loc = left;
    }
```

```
                }
  }
return loc;
}
void quick_sort(int a[], int beg, int end)
{  intloc;
   if(beg<end)
     {
loc = partition(a, beg, end);
quick_sort(a, beg, loc-1);
quick_sort(a, loc+1, end);
      }
}
```

Programming Example of Quick sort with recursive algorithm:

```
/*Program of sorting using quick sort through recursion*/
#include<stdio.h>
#include<conio.h>
void display(int[],int,int);
int quick(int[],int,int);
enum bool {FALSE,TRUE};

int main()
{       int array[6]={29,12,38,20,27,47};
        int n=6,i;
        clrscr();
        printf("The given Unsorted list is :\n");
        display(array,0,n-1);
        printf("\n");
        quick(array,0,n-1);  //Calling Quick sort
        function printf("Sorted list is :\n");
        display(array,0,n-1);
        printf("\n");
return 0;
}
int quick(intarr[],intlow,int up)
{       intpiv,temp,left,right;
        enum bool pivot_placed=FALSE;
        left=low;
        right=up;
        piv=low; /*Taking pivot as first element */
        if(low>=up)
                return 0;
        printf("Sublist : ");
        display(arr,low,up);
        /*Loop to set pivot on its proper place */
        while(pivot_placed==FALSE)
```

```
        {
/*Compare from Right to Left  */
                while( arr[piv]<=arr[right] &&piv!=right )
                        right=right-1;
                if( piv==right )
                pivot_placed=TRUE;
                if( arr[piv] >arr[right] )
                {
                        temp=arr[piv];
                        arr[piv]=arr[right];
                        arr[right]=temp;
                        piv=right;
                }
/*Compare from Left to Right */
                while( arr[piv]>=arr[left] && left!=piv )
                        left=left+1;
                if(piv==left)
                        pivot_placed=TRUE;
                if( arr[piv] <arr[left] )
                {       temp=arr[piv];
                        arr[piv]=arr[left];
                        arr[left]=temp;
                        piv=left;
                }
        }
        printf("-> Pivot Placed is %d -> ",arr[piv]);
        display(arr,low,up);
        printf("\n");
        quick(arr,low,piv-1);
        quick(arr,piv+1,up);
return 0;
}
void display(intarr[],intlow,int up)
{       inti;
        for(i=low;i<=up;i++)
                printf("%d ",arr[i]);
}
```

10.10 MERGE SORT

The basic concept of merge sort is divides the list into two smaller sub-lists of approximately equal size. Recursively repeat this procedure till only one element is left in the sub-list. After this, various sorted sub-lists are merged to form sorted parent list. This process goes on recursively till the original sorted list arrived.

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Merge sort algorithm focuses on two main concepts to improve its performance (running time):

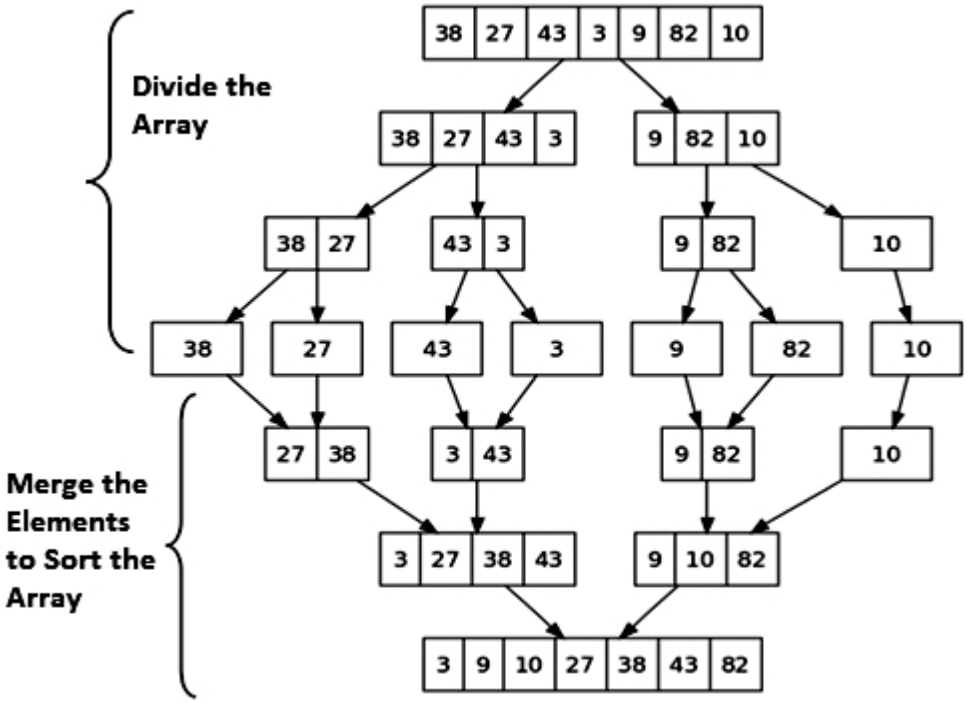
- A smaller list takes fewer steps and thus less time to sort than a large list.
- As number of steps is relatively less, thus less time is needed to create a sorted list from two sorted lists rather than creating it using two unsorted lists.

The basic steps of a merge sort algorithm are as follows:

- If the array is of length 0 or 1, then it is already sorted.
- Otherwise, divide the unsorted array into two sub-arrays of about half the size.
- Use merge sort algorithm recursively to sort each sub-array.
- Merge the two sub-arrays to form a single sorted list.

Example: Let us take a list of unsorted elements which are as under

38	27	43	3	9	82	10



Resultant Sorted array is:

3	9	10	27	38	42	82

This merge sort algorithm will use a function MERGE which combines the sub-arrays to form a sorted array. While the merge sort algorithm recursively divides the list into smaller lists, the merge algorithm conquers the list to sort the elements in individual lists. Finally, the smaller lists are merged to form one original list which will be sorted list.

Algorithm for Merge Sort:

MERGE (Array, Beg, Mid, End)

Step 1: [Initialize] Set I= Beg, J= Mid+1, idx = 0

Step 2: Repeat While (I <= Mid) AND (J<=End)

If Array[I]< Array[J]

Set Temp[idx]= Array[I]

Set I = I + 1

Else

Set Temp[idx] = Array[J]

Set J = J + 1

[End of If]

Set idx = idx + 1

[End of Loop]

Step 3: [Copy the remaining elements of right sub-Array, If any]

If I > Mid

Repeat While J<= End

Set Temp[idx] = Array[J]

Set idx = idx + 1, Set J = J + 1

[End of Loop]

[Copy the remaining elements of left sub-Array, If any]

Else

Repeat WhileI<= Mid

Set Temp[idx] = Array[I]

Set idx= idx + 1, Set I = I + 1

[End of Loop]

[End of If]

Step 4: [Copy the contents of Temp back to Array] Set K=

Step 5: Repeat WhileK<idx

Set Array [K] = Temp[K]

SetK = K + 1

[End of Loop]

Step 6: End

Merge_Sort(Array, Beg, End)

Step 1: IF Beg< End

SET Mid= (Beg+ End)/2

CALL Merge_Sort (Array, Beg, Mid)

```
CALL Merge_Sort (Array, Mid+1, End)

Merge (Array, Beg, Mid, End)

[End of If]

Step 2: End
```

The running time of merge sort in the average case and the worst case can be g iven a s O (n l og n). A lthough m erge s ort ha s a n opt imal t ime complexity, i t ne eds a n additional s pace o f O (n) f or t he t emporary array TEMP.

Programming Example:

```
/* Program of sorting using merge sort */
#include<stdio.h>
#include<conio.h>
void mergesort(int a[],inti,int j);
void merge(int a[],int i1,int j1,int i2,int j2);
int main()
{       int a[10]={38, 27, 43, 3, 9, 82, 10, 28, 89,74};
        inti,n=10;
        clrscr();
        printf("The given unsorted list is as under:\n");
        for(i=0;i<n;i++)
                printf("%d ",a[i]);
        mergesort(a,0,n-1);
        printf("\nSorted array is :");
        for(i=0;i<n;i++)
                printf("%d ",a[i]);
        return 0;
}
void mergesort(int a[],inti,int j)
{       int mid;
        if(i<j)
        {       mid=(i+j)/2;
                mergesort(a,i,mid);             //left recursion
                mergesort(a,mid+1,j); //right recursion
                merge(a,i,mid,mid+1,j);         //merging o f tw o s orted s ub-
arrays
        }
}
void merge(int a[],int i1,int j1,int i2,int j2)
{       int temp[50];   //array used for merging
```

```
        inti,j,k;
        i=i1;   //beginning of the first list
        j=i2;   //beginning of the second list
        k=0;
        while(i<=j1 && j<=j2)          //while elements in both lists
        {       if(a[i]<a[j])
                        temp[k++]=a[i++];
                else
                        temp[k++]=a[j++];
        }
        while(i<=j1)   //copy remaining elements of the first list
                temp[k++]=a[i++];

        while(j<=j2)   //copy remaining elements of the second list
                temp[k++]=a[j++];
        //Transfer elements from temp[] back to a[]
        for(i=i1,j=0;i<=j2;i++,j++)
                a[i]=temp[j];
}
```

10.11 SHELL SORT

D. L. Shell proposed an improvement on insertion sort in 1959 named after him as Shell Sort. It is a generalization of insertion sort. Shell sort is considered an improvement over insertion sort as it compares elements separated by a gap of several positions while in Insertion sort compares with adjacent item. This Shell sort enables the element to take a big gap. In Shell sort, elements are sorted in multiple passes and in each pass, data are taken from big gap to smaller gap sizes. And finally in last step, it will work like plain insertion sort. As we reach towards last step, the items are mostly sorted which results in good performance.

Let us take an example to understand the concept of Shell Sort. Suppose we have an array with following elements.

60	**16**	**4**	**87**	**78**	**33**	**51**	**42**	**69**	**24**	**19**	**6**	**38**	**56**	**30**

Step 1. Make arrangement of elements in two rows in the form of a table and sort each columns as under.

									→								
Row 1	**60**	**16**	**4**	**87**	**78**	**33**	**51**	**42**		**60**	**16**	**4**	**6**	**38**	**33**	**30**	**42**
Row 2	**69**	**24**	**19**	**6**	**38**	**56**	**30**			**69**	**24**	**19**	**87**	**78**	**56**	**51**	

Now the elements of the array will be as under after merging both sorted rows.

60	16	4	6	38	33	30	42	69	24	19	87	78	56	51

Step 2. Now repeat step 1 again with small column gap size as under in three rows.

Row 1	**60**	**16**	**4**	**6**	**38**		**19**	**16**	**4**	**6**	**24**
Row 2	**33**	**30**	**42**	**69**	**24**	→	**33**	**30**	**42**	**56**	**38**
Row 3	**19**	**87**	**78**	**56**	**51**		**60**	**87**	**78**	**69**	**51**

Now the elements of the array will be as under after merging sorted column

19	16	4	6	24	33	30	42	56	38	60	87	78	69	51

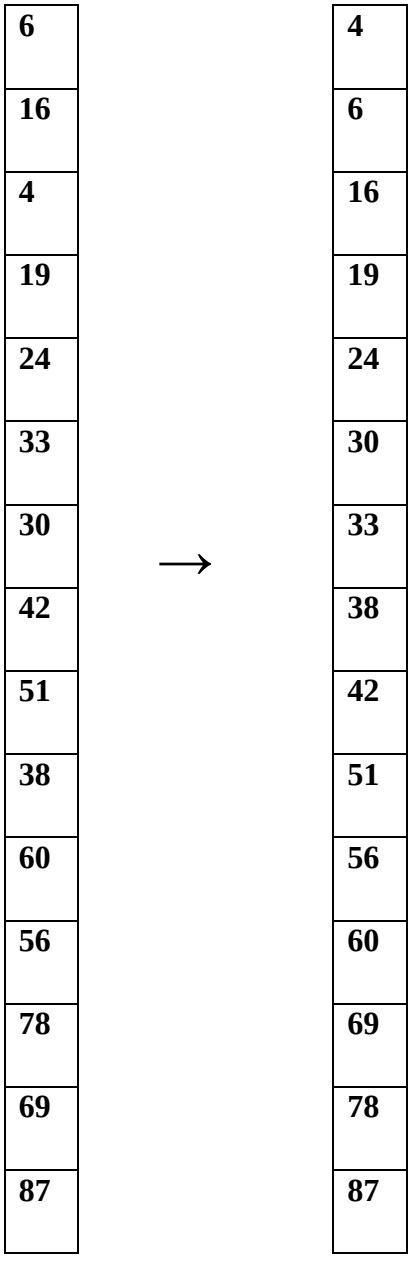
Step 3. Now repeat previous step again with small column gap size as under in five rows.

Row 1	**19**	**16**	**4**		**6**	**16**	**4**
Row 2	**6**	**24**	**33**	⟶	**19**	**24**	**33**
Row 3	**30**	**42**	**56**		**30**	**42**	**51**
Row 4	**38**	**60**	**87**		**38**	**60**	**56**
Row 5	**78**	**69**	**51**		**78**	**69**	**87**

Now the elements of the array will be as under after merging sorted column.

6	16	4	19	24	33	30	42	51	38	60	56	78	69	87

Step 4. Now repeat previous step again with single column and sort it.



Finally, the elements of the array can be given as:

4	6	16	19	24	30	33	38	42	51	56	60	69	78	87

Algorithm for shell sort:

The algorithm to sort an array of elements using shell sort is as under in various steps. We shall use multiple passes to sort the array Arr. After every pass, reduce the gap(See the number of columns) by a factor of half as shown in Step 4. For each iteration in for loop in Step 5, we swap the array values accordingly with smaller one if required.

```
SHELL_SORT(Arr, n)

Step 1: SET FLAG=1, GAP =N

Step 2: Repeat Steps 3 to 6 while FLAG=1 OR GAP > 1

Step 3:         SET FLAG = 0

Step 4:         SET GAP = (GAP + 1) / 2

Step 5:         Repeat Step 6 for I = to I < (N- GAP)

Step 6:         IF Arr[I+ GAP]>Arr[I]

                        SWAP Arr[I+ GAP], Arr[I]

                        SET FLAG =0

Step 7: END
```

Programming Example:

```
/* Program of sorting using shell sort */
#include <stdio.h>
void main()
{       inti,j,k,n=15,incr;
        intarr[15]={60,16,4,87,78,33,51,42,69,24,19,6,38,56,30};
        printf("The goven Unsorted list is :\n");
        for (i = 0; i< n; i++)
                printf("%d ", arr[i]);
        printf("\nEnter maximum increment (odd value) : ");
        scanf("%d",&incr);
        /*Shell sort*/
        while(incr>=1)
```

```
        {       for(j=incr;j<n;j++)
                {       k=arr[j];
                        for(i = j-incr; i>= 0 && k <arr[i]; i = i-incr)
                                arr[i+incr]=arr[i];
                        arr[i+incr]=k;
                }
                printf("Increment=%d \n",incr);
                for (i = 0; i< n; i++)
                        printf("%d ", arr[i]);
                printf("\n");
                incr=incr-2; /*Decrease the increment*/
        }/*End of while*/
printf("Sorted list is :\n");
        for (i = 0; i< n; i++)
                printf("%d ", arr[i]);
        printf("\n");
getch();
}
```

1.12 RADIX SORT

Radix sort is a linear sorting algorithm for integers and uses the concept of sorting names in alphabetical order. When we have a list of sorted names, the radix is 26 (or 26 buckets) because there are 26 letters in the English alphabet. So radix sort is also known as bucket sort. Observe that words are first sorted according to the first letter of the name. That is, 26 classes are used to arrange the names, where the first class stores the names that begin with A, the second class contains the names with B, and so on.

When radix sort is used on integers, sorting is done on each of the digits in the number. The sorting procedure proceeds by sorting the least significant to the most significant digit. While sorting the numbers, we have ten buckets, each for one digit (0, 1, 2,…, 9) and the number of passes will depend on the length of the number having maximum number of digits. If numbers are of two digit, then 2 passes will be there and if the numbers are of three digits then 3 passes will be there.

Example 1: Let us take an array of 12 numbers in unsorted order and sort them using radix sort.

42 20 64 51 34 70 31 16 15 12 19 33

In first element 42, unit digit is 2. So in first pass, the unit digit (first from last) will be sorted.

Pass 1 for unit digit

Numbers	0	1	2	3	4	5	6	7	8	9
42			42							
20	20									
64					64					
51		51								
34					34					
70	70									
31		31								
16							16			
15						15				
12			12							
19										19
33				33						

After Pass 1, the numbers are as follow

20 70 51 31 42 12 33 64 34 15 16 19

Pass 2 for ten's digit :

Numbers	0	1	2	3	4	5	6	7	8	9
20			20							
70								70		
51						51				
31				31						
42					42					
12		12								
33				33						
64							64			
34				34						
15		15								
16		16								
19		19								

After Pass 2, the numbers are as follow which are sorted.

12 15 16 19 20 31 33 34 42 51 64 70

Example 2: Let us ta ke another exa mple of three digit nu mbers t o sort using Radix sort. We have an array of 11 numbers as under.

342	651	921	120	564	469	552	805	908	526	443

Pass 1 for Unit Digit

Number	0	1	2	3	4	5	6	7	8	9
342			342							
651		651								
928									928	
120	120									
564					564					
469										469
552			552							
805						805				
907								907		
526							526			
443				443						

After Pass 1 the array values will be like this

120	651	343	552	443	564	805	526	907	928	469

Pass 2 for ten's Digit

Number	0	1	2	3	4	5	6	7	8	9
120			120							
651						651				
343					343					
552						552				
443					443					
564							564			
805	805									
526			526							
907	907									
928			928							
469							469			

After Pass 2 the array values will be like this

805	907	120	526	928	343	443	651	552	564	469

Pass 3 for Hundred Digit

Number	0	1	2	3	4	5	6	7	8	9
805									808	
907										907
120		120								

526						526				
928										928
343				343						
443					443					
651							651			
552						552				
564						564				
469					469					

After Pass 3 the array values will sorted as below

120	343	443	469	526	552	564	651	808	907	928

Algorithm for Radix Sort

Radix Sort (ARR, N)

Step 1: Find the largest number in ARR as LARGE

Step 2: [INITIALIZE] SET NOP= Number of digits in LARGE

Step 3: SET PASS = 0

Step 4: Repeat Step 5 while PASS <= NOP-1

Step 5: SET I= 0 and INITIALIZE buckets

Step 6: Repeat Steps7to9 while I<N-1

Step 7: SET DIGIT = digit at PASSth place in A[I]

Step 8: Add A[I] to the bucket numbered DIGIT

Step 9: INCEREMENT bucket count for bucket numbered DIGIT

[END OF LOOP]

Step 10: Collect the numbers in the bucket

[END OF LOOP]

Step 11: END

Programming Example:

```
/*Program of sorting using Radix sort */
#include <stdio.h>
#include <conio.h>
#define size 10
int largest(intarr[], int n);
void radix_sort(intarr[], int n);
int main()
{  intarr[12]={42,20,64,51,34,70,31,16,15,12,19,33};
inti, n=12;
clrscr();
printf("\n The given unsorted list is as under \n");
  for(i=0;i<n;i++)
printf("%d  ",arr[i]);
printf("\n");
radix_sort(arr, n);
printf("\n The sorted array is: \n");
  for(i=0;i<n;i++)
printf("%d  ",arr[i]);
return 0;
}
int largest(intarr[], int n)
{
int large=arr[0], i;
  for(i=1;i<n;i++)
```

```
  {
   if(arr[i]>large)
   large = arr[i];
  }
  return large;
}
void radix_sort(intarr[], int n)
{
int bucket[size][size], bucket_count[size];
inti, j, k, remainder, NOP=0, divisor=1, large, pass;
  large = largest(arr, n);
  while(large>0)
  {   NOP++;
      large/=size;
  }
  for(pass=0;pass<NOP;pass++) // Initialize the buckets
  {    for(i=0;i<size;i++)
bucket_count[i]=0;
       for(i=0;i<n;i++)
        {
            // sort the numbers according to the digit at passth place
               remainder = (arr[i]/divisor)%size;
               bucket[remainder][bucket_count[remainder]] = arr[i];
bucket_count[remainder] += 1;
         }
   // collect the numbers after PASS pass
i=0;
   for(k=0;k<size;k++)
   {
      for(j=0;j<bucket_count[k];j++)
        { arr[i] = bucket[k][j];
i++;
          }
        }
   divisor *= size;
  }
```

```
}
```

1.13 SUMMARY

Now we have understand all the sorting algorithms with their worst, average and best cases. But one thing should be very clear that, no sorting technique is best. It should depend on the situation of the data. Choice of algorithm should depends on the order of the data and storage location of it. Now let us summarize all the sorting algorithms with respect to their best, average and best case behavior in term of O notation.

Sorting Technique	Best Case	Average case	Worst case
Selection Sort	$O(n^2)$	$O(n^2)$	$O(n^2)$
Bubble Sort	O(n)	$O(n^2)$	$O(n^2)$
Insertion Sort	O(n)	$O(n^2)$	$O(n^2)$
Heap Sort	O(n log n)	O(n log n)	O(n log n)
Quick Sort	$O(n^2)$	O(n log n)	O(n log n)
Merge Sort	O(n log n)	O(n log n)	O(n log n)
Shell Sort	-	-	-
Radix Sort	$O(n^2)$	O(n log n)	O(n log n)

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SELF-EVALUATION

1. Write a r ecursive "C " f unction to i mplement b inary s earch an d compute its time complexity.
2. Sort the sequence 3, 1, 4, 5, 9, 2, 6, 5 using insertion sort.
3. Explain sequential search with suitable example.
4. What do you mean by sorting? Explain bubble sort with example.
5. Sort the sequence 33, 11, 64, 58, 94, 62, 76, 15 using quick sort
6. Sort the sequence 43, 18, 94, 28, 64, 92, 36, 75, 82, 99using heap sort
7. Sort t he s equence 93, 4 8, 74, 29, 34, 97, 39, 25, 42, 19 us ing selection sort
8. Sort the sequence 63, 98, 14, 28, 54, 72, 16, 17, 22, 79using merge sort
9. Sort the sequence using Radix sort

   103, 108, 924, 248, 664, 912, 736, 375, 182, 399
10. Show how heap sort processes the input values as under.

    142, 543, 123, 65, 453, 879, 572, 434, 111, 242, 811, 102.

Multiple Choice Questions

1. What is the worst-case time for serial search finding a s ingle item in an array?

   A. Constant time B. Quadratic time

   C. Logarithmic time **D. Linear time**
2. What is the worst-case time for binary search finding a single item in an array?

   A. Constant time **B. Quadratic time**

   C. Logarithmic D. Linear time
3. Which searching can be performed recursively?

   A. Linear **B. both**

   C. Binary search D. none
4. Which searching can be performed iteratively? [B]

   A. linear search **B. Both**

   C. Binary search D. none

5. In a selection sort of n elements, how many times is the swap function called in the complete execution of the algorithm?

A. 1 **B. n^2**

C. n – 1 D. n log n

6. Selection sort and quick sort both fall into the same category of sorting algorithms. What is this category?

A. O(n log n) sorts

B. Interchange sorts

C. Divide-and-conquer sorts

D. Average time is quadratic

7. When is insertion sort a good choice for sorting an array?

A. Each component of the array requires a large amount of memory

B. The array has only a few items out of place

C. Each component of the array requires a small amount of memory

D. The processor speed is fast

8. What is the worst-case time for quick sort to sort an array of n elements?

A. O(log n) B. O(n)

C. O(n log n **D. $O(n^2)$**

9. What is the worst-case time for heap sort to sort an array of n elements?

A. O(log n B. O(n)

C. O(n log n) D. $O(n^2)$

10. A min heap is the tree structure where smallest element is available at the

A. leaf **B. root**

C. intermediate parent D. Any where

11. Which design algorithm technique is used for quick sort?

A. Divide and conqueror B. greedy

C. backtrack D. dynamic programming

12. Which among the following is fastest sorting technique (for unordered data)

A. Heap sort B. Selection Sort

C. Quick Sort D. Bubble sort

13. In which searching technique elements are eliminated by half in each pass.

A. Linear search **B. Binary search**

C. Both D. none

14. Binary search algorithm performs efficiently on a

A. linked list B. both

C. Array D. None

14. Heap is a good data structure to implement

A. priority Queue B. Deque

C. linear queue D. None

True or False

1. Binary search is also called sequential search.
2. Linear search is performed on a sorted array.
3. For insertion sort, the best case occurs when the array is already sorted.
4. Selection sort has a linear running time complexity.
5. The running time of merge sort in the average case and the worst case is O(n log n).
6. The worst case running time complexity of quick sort is O(n log n).
7. Heap sort is an efficient and a stable sorting algorithm.
8. External sorting deals with sorting the data stored in the computer's memory.
9. Insertion sort is less efficient than quick sort, heap sort, and merge sort.
10. The average case of insertion sort has a quadratic running time.
11. The partitioning of the array in quick sort is done in O(n) time.
12. In internal sorting methods, it is necessary for all data should reside in main memory.

Answer

1. False	2. False	3. False	4. False	5. True	6. False
7.True	8. False	9. True	10. True	11. True	12. True

UNIT-11 HASHING

Structure:

11.0 INTRODUCTION

The searching techniques that we have discussed in the previous unit 10 are based on comparison of values with each other. In sequential search, we have to search from the beginning and move up to last element, so all items are compared with the searched items depend on its location in the array. In binary search, less comparison are there with respect to sequential search as on each step list is divided in two halves. So there is less comparison. So there is a need where we have to do minimum comparisons so that complexity could be reduced. So now our need is to search the element in constant times and less key comparison should be involved. Finally, the main objective is that, how to reduce the number of comparisons in order to find the appropriate record within minimum time and minimum comparison.

11.1 OBJECTIVE

After reading this unit the learner is able to do the following task.

- Understand the concept of hashing and its need.
- Understand hash function and its methods
- Understand Collision and its resolution strategies
- Understand hash table and its implementation

11.2 Hashing

Hashing is a technique that is used to uniquely identify a specific object from a group of similar objects. Some examples of how hashing is used in our lives include:

- In universities, each student is assigned a unique roll number that can be used to retrieve information about them.
- In libraries, each book is assigned a unique number that can be used to determine information about the book, such as its exact position in the library or the users it has been issued to etc.

In both these examples the students and books were hashed to a unique number.Assume that you have an object and you want to assign a key to it to make searching easy. To store the key/value pair, you can use a simple array like a data structure where keys (integers) can be used directly as an index to store values. However, in cases where the keys are large and cannot be used directly as an index, you should use hashing.

In hashing, large keys are converted into small keys by using hash functions. The values are then stored in a data structure called hash table. The idea of hashing is to distribute entries (key/value pairs) uniformly across an array. Each element is assigned a key (converted key). By using that key you can access the element in O(1) time. Using the key, the algorithm (hash function) computes an index that suggests where an entry can be found or inserted.

Hashing is implemented in two steps: In first step, the element is converted into an integer by using a hash function. This element can be used as an index to store the original element, which falls into the hash table. In second step, the element is stored in the hash table where it can be quickly retrieved using hashed key.

hash = hashfunc(key)

index = hash % array_size

In this method, the hash is independent of the array size and it is then reduced to an index (a number between 0 and N – 1(N is size of array) by using the modulo operator (%).

Let us take an example to explain. In a university, there are 100 faculties, and each faculty has assigned aTeacher_ID in the range 0–99. To store the records in an array, each faculty Teacher_ID acts as an index into the array where the faculty's record will be stored as shown in Figure 1 below. In this case, we can directly access the record of any faculty, once we know his Teacher_ID, because the array index is the same as the Teacher_ID number. But practically, this implementation is hardly feasible.

Key	Array of Faculty's Records
Key 0 → [0]	Faculty record with Teacher_ID 0
Key 1 → [1]	Faculty record with Teacher_ID 1
Key 2 → [2]	Faculty record with Teacher_ID 2
----------	----------
----------	----------
Key 98 → [98]	Faculty record with Teacher_ID 98
Key 99 → [99]	Faculty record with Teacher_ID 99

Figure 1 : Faculties records with two digit Teacher_id.

Let us take a similar case of university which has fivedigitTeacher_ID as the primary key. In this case,key values will range from 00000 to 99999. If we want to use the same technique as above, we need an array of size 100,000, of which only 100 elements will be used. This is illustrated in Figure 2.

Key	Array of Faculty's Records
Key 00000 → [0]	Faculty record with Teacher_ID 0
Key 00001 → [1]	Faculty record with Teacher_ID 1
Key 00002 → [2]	Faculty record with Teacher_ID 2
----------	----------
Key n → [n]	Faculty record with Teacher_ID n
----------	----------
Key 99998 → [99998]	Faculty record with Teacher_ID 99998
Key 99999 → [99999]	Faculty record with Teacher_ID 99999

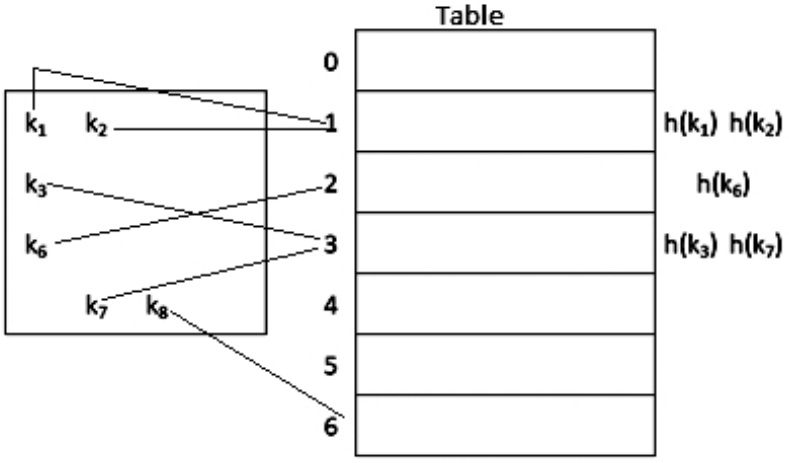
Figure 2 : Faculties records with five digitTeacher_id.

It is obvious to waste a lot of memory storage space just to ensure that each employee's record is unique and it is on predictable location.

So if we use a two-digit primary key (Teacher_ID) or a five-digit key, we have only 100 faculties in the university. For this we will be using only 100 locations in the array. Therefore, in order to keep the array size down to the size that we will actually be using (100 elements), another good option is to use just the last two digits of the key to identify each employee. For example, the faculty with Teacher_ID79439 will be stored in the element of the array with index 39. Similarly, the faculty with Teacher_ID12345 will have his record stored in the array at the 45th location.

11.3 HASH TABLE

Hash table is a data structure in which keys are mapped to array positions by a hash function which is arranged in the form of an array that is addressed via a hash function. The hash table is divided into a number of buckets and each bucket is in turn capable of storing a number of records. Thus we can say that a bucket has number of slots and each slot is capable of holding one record.



The time required to locate any element in the hash table is O (1). It is constant and it is not depend on the number of data elements stored in the table. Now question is how we map the number of keys to a particular location in the hash table i.e., h (k). It is computed using the hash function.

11.4 HASH FUNCTION

Hash function is just a mathematical calculation to calculate the key used as an index. The basic idea in hashing is the transformation of a key into the corresponding location in the hash table. This is done by a hash function. To achieve a good hashing mechanism, it is important to have a good hash function with the following basic requirements:

- **Easy to compute**: It should be easy to compute and must not become an algorithm in itself.
- **Uniform distribution**: It should provide a uniform distribution across the hash table and should not result in clustering.
- **Less collision**: Collisions occur when pairs of elements are mapped to the same hash value. These should be avoided.

Sometimes, this hash function may not yield distinct values; it is possible that two different keys K_1 and K_2 will yield the same hash address. This situation is called **Hash collision**.

Note: Irrespective of how good a hash function is, collisions are bound to occur. Therefore, to maintain the performance of a hash table, it is important to manage collisions through various collision resolution techniques.

There are three different types of Hash functions. There are two points to be in mind while choosing a function $H : K \rightarrow M$. The first one is that, it should be easy and fast to calculate. Another one is that, it should always give different location (unoccupied) locations in the hash table to avoid collisions.

These hash functions are as under:

(i) Truncate Method

(ii) Division reminder method

(iii) Mid square method –

(iv) Folding method.

(v) For Floating point number

(vi) For Strings

11.4.1 Truncate Method

This is one of the easiest method for calculating the key value for hash function. In this method, we take only a part of the key as address. It could be from leftmost digits or rightmost digits.

EXAMPLE 1. Calculate the hash values or keys of following 8 digit numbers for hash table of size 100.

76895534 78933524 93592415 18935445

SOLUTION.

Now as the table size is 100, so take 2 rightmost digit for getting the hash table address as under.

34 24 15 45

Key	Rightmost 2 digits
76895534	34
78933524	24
93592415	15
18935445	45

Now we have taken right two digit as address, but there are a lot of chances that collision can occur because last two digits can be same in many numbers.

11.4.2 DIVISION REMINDER METHOD

In this division reminder method of hash function, key k is divided by a number m larger than the number n of keys in k and the reminder of this division is taken as index into the hash table, i.e.,

$$h(k) = k \textbf{ Mod } m$$

The number m should be usually a prime number or a number without small divisors, so that it minimizes the number of collision possibilities.

The above hash function will map the keys in the range 0 to m —1 and is acceptable in C/C++. But if we want the hash addresses to range from 1 to m rather than from 0 to m —1 we use the formula

$$h(k)=k \textbf{ Mod } m+1$$

EXAMPLE 2. Calculate the hash values of keys 1234 and 5462.

Solution: Setting M = 97, hash values can be calculated as:

h (1234) = 1234 % 97 = 70

h (5642) = 5642 % 97 = 16

Consider a hash table with 7 slots i.e.,m = 7, then hash function h (k) = k mod in will map the key 169 to slot 1 since

h (169) = 169 mod 7 =1

similarly,

h (130) = 130 mod 7 = 4

is mapped to slot 4.

11.4.3 MID SQUARE METHOD

In the mid square method the key is first squared. Therefore the hash function is defined by

$$h(k)=p$$

where p is obtained by deleting digits from both sides of k^2. To properly implement this the same position of k^2 must be used for all the keys.

EXAMPLE 3. Consider a hash table with 50 slots i.e., m = 50 and key values k= 1632, 1739,3123.

SOLUTION.

k	1632	1739	3123
k^2	266**34**24	302**41**21	975**31**29
h(k)	34	41	31

The hash values are obtained by taking the fourth and fifth digits counting from right.

11.4.4 FOLDING METHOD

In folding method the key, k is partitioned into a number of parts k_1, k_2….k_r where each part, except possibly the last, has the same number of digits as the required address: Then the parts are added together, ignoring the last carry i.e.,

$$H(k)=k_1+k_2+\ldots\ldots\ldots +k_r$$

wherethe leading-digits carries, if any are ignored.

EXAMPLE 4. Consider a hash table with 100 slots i.e., m = 100 and key values k = 7325, 76321, 1623, 7613.

SOLUTION.

k	Parts	Sum of parts	h (k)
7325	73, 25	98	98
76321	76, 32, 1	109	09
1623	16, 23	39	39
7613	76, 13	89	89

11.4.5 FOR FLOATING POINT NUMBER

The approach for hash address for floating point numbers is something different but also needs modulus operation within the range of hash table. There are following steps for calculating the hash values for such floating point numbers.

1. Take fractional part of the key.
2. Multiply this part with size of hash table array.
3. Now take integer part from it as the result of hash address of that key.

Example 5 : Calculate the hash values of keys 123.6721, 970.663, 123.0558 and 679.99156 with table size 99.

Solution :

Let us take the fractional part of the keys and multiply with table size 99.

$0.6721 \times 99 = 66.5379$

$0.663 \times 99 = 65.637$

$0.0558 \times 99 = 5.5242$

$0.99156 \times 99 = 98.16444$

Now the hash address will be integer part of these numbers.

H (66.5379) = 66

H (65.637) = 65

H (5.5242) = 5

H (98.16444) = 98

11.4.6 FOR STRINGS

In many cases strings are used as the key which could be alphabetic or alphanumeric. We can see it in English dictionary very frequently. We can take ASCII value of each character and sum all values then take modulus by the table size to calculate the key for hash table.

Let us take an example.

Suppose we have a hash table of size 99 and the key is “Manisha”. Now we have to calculate the hash value for it.

First sum up all the ASCII values respected to each character in the key as under

Manisha	M	a	n	i	s	h	a	**Total**
	77	97	110	105	115	104	97	**705**

So

H(Manisha) = 707%99=12

Now key "Manisha" can be mapped to 12^{th} location in the hash table.

11.5 RESOLVING COLLISION

Hash collision is the process in which more than one key is mapped to the same memory location in the table. For example, if we are using the division reminder hashing with following hash-function

h (k) = k % 7 then key = 8 and key =15 both mapped to the same location of the table i.e., one

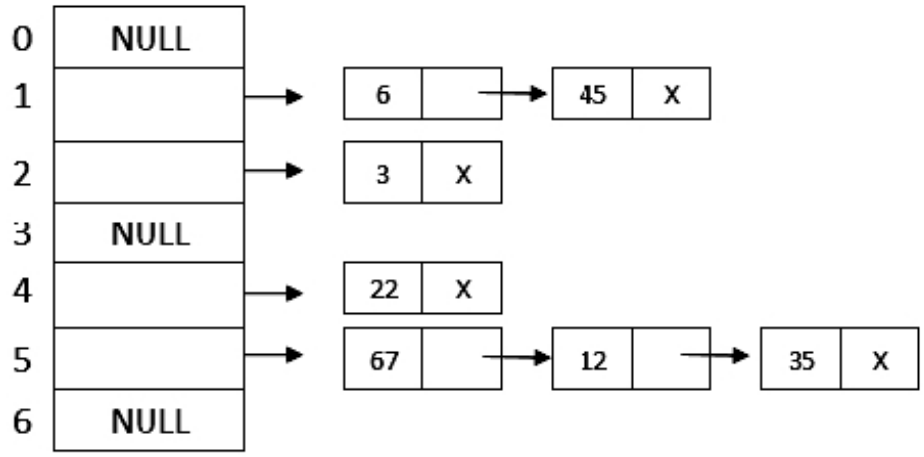
h (k) = 8 % 7 = 1

h (k) = 15 % 7 = 1

Both key will store at same location in the hash table and collision will occur.

11.5.1 COLLISION RESOLUTION BY SEPARATE CHAINING (OPEN HASHING)

This method maintains a chain of all elements which have same address. In this method all the elements where keys hash to the same hash-table slot are put in a one array of pointers or a linked list. Therefore, the slot i in the hash table contains a pointer to the head of the linked list of all the elements that hash to value i. If there is no such elements that hash to value i, the slot contains NULL value.



11.5.2 Collision Resolution by Open Addressing (Closed Hashing)

In open addressing the keys to be hashed is to put in the separate location of the hash table. Eachlocation contains some key or the some other character to indicate that the particular location is free. In this method to insert key into the table we simply hash the key using the hash function. If the space is available, then insert the key into the hash table location otherwise; search the location in the forward direction of the table, to find the slot in a systematic manner. The process of finding the slot in the hash table is called probing

11.5.2.1 LINEAR PROBING

This hashing technique finds the hash key value through hash function and maps the key on particular position in hash table. In case if key has same hash address then it will find the next empty position in the hash table. We take the hash table as circular array. So if table size is N then after N-1 position it will search from 0th position in the array.

The linear probing uses the following hash function

$$h(k, i) = [h'(k) + i] \bmod n \text{ for } i = 0, 1, 2, \ldots n-1$$

where n is the size of the hash table and $h'(k) = k \bmod n$ the basic hash function and i is the probe number.

Let us take some elements and the table size is 11.

30, 19, 44, 11, 37, 24, 47

0	44
1	11
2	
3	47
4	37
5	26
6	
7	
8	30
9	19
10	

H(30) = 30%11 = 8

H(19) = 19%11 = 8

H(44) = 44%11 = 0

H(11) = 11%11 = 0

H(37) = 37%11 = 4

H(26) = 26%11 = 4

H(47) = 47%11 = 3

Now the first 30 will be inserted at the 8^{th} position in the array. Next 19 will also on same hash, address 8^{th}, but it is already occupied, so it will search for the next free place which is 9^{th} position. Similarly 44 and 11 also has same hash address i.e. 0^{th} position, so after insertion of 44 at 0^{th} position, 11 will be on next position i.e. 1^{st} position. Similarly key value 37 and 26 has same, so that will store at 4^{th} and 5^{th} position. In last key 47 will be at 3^{rd} position.

The main disadvantage of the linear probing technique is clustering problem. When half of the table is full then it is difficult to find empty position in hash table in case of collision. Searching will also become slow because it will go for linear searching.

11.5.2.2 QUADRATIC PROBING

The main disadvantage of linear probing is clustering problem. Suppose hash address is k then in the case of collision linear probing search the location k, k+1, k+2 (%SIZE). Here in quadratic probing it search the location $(k+i^2)$%SIZE (for i=1,2,3,4…….). So it will search the locations k+1, k+4, k+9…… So it will decrease the problem of clustering but this technique cannot search all the locations. If hash table size is prime then it will search at least half of the locations of the hash table. Let us take table size 11 and apply this technique with following elements-

0	10
1	
2	46
3	54
4	
5	
6	
7	29
8	18
9	
10	43

H(29) = 29%11 = 7

H(18) = 18%11 = 7

H(43) = 43%11 = 10

H(10) = 10%11 = 10

H(46) = 46%11 = 2

H(54) = 54%11 = 10

As well as we insert the element 43 at 10^{th} position in table, the element 10 will search the empty position at (k +1) %11 = 0th position which is empty. When we insert 54 then after getting collision first it will search the next position (k+1) %11 = 0th position which is already occupied, so it will again search the next position (k+4) % 11 = 3 position which is empty so 54 will be inserted at that position

11.5.2.3 DOUBLE HASHING

Double hashing is similar to linear probing and the only difference is the interval between successive probes. Here, the interval between probes is computed by using two hash functions.

Let us say that the hashed index for an entry record is an index that is computed by one hashing function and the slot at that index is already occupied. You must start traversing in a specific probing sequence to look for an unoccupied slot. The probing sequence will be:

index = (index + 1 * indexH) % hashTableSize;
index = (index + 2 * indexH) % hashTableSize;

and so on…

Here, **indexH** is the hash value that is computed by another hash function.

11.6 SOME APPLICATIONS OF HASH TABLES

Database systems: Specifically, those that require efficient random access. Generally, database systems try to optimize between two types of access methods: sequential and random. Hash tables are an important part

of efficient random access because they provide a way to locate data in a constant amount of time.

Symbol tables: The tables used by compilers to maintain information about symbols from a program. Compilers access information about symbols frequently. Therefore, it is important that symbol tables be implemented very efficiently.

Data dictionaries: Data structures that support adding, deleting, and searching for data. Although the operations of a hash table and a data dictionary are similar, other data structures may be used to implement data dictionaries. Using a hash table is particularly efficient.

Network processing algorithms: Hash tables are fundamental components of several network processing algorithms and applications, including route lookup, packet classification, and network monitoring.

Associative arrays: Hash tables are commonly used to implement many types of in-memory tables. They are used to implement associative arrays (arrays whose indices are arbitrary strings or other complicated objects).

Caches: Hash tables can be used to implement caches i.e. auxiliary data tables that are used to speed up the access to data, which is primarily stored in slower media.

Object representation: Several dynamic languages, such as Perl, Python, JavaScript, and Ruby use hash tables to implement objects.

Browser Cashes: Hash tables are used to implement browser cashes.

11.7 SUMMARY

Hashing is the process of mapping large amount of data item to smaller table with the help of hashing function. Hashing is also known as **Hashing Algorithm**.It is a technique to convert a range of key values into a range of indexes of an array. It is used to facilitate the next level searching method when compared with the linear or binary search.

Hash table or hash map is a data structure used to store key-value pairs.It is a collection of items stored to make it easy to find them later.It uses a hash function to compute an index into an array of buckets or slots from which the desired value can be found.It contains value based on the key.

Hash Function is a fixed process which converts a key to a hash key to store in array.This function takes a key and maps it to a value of a certain length which is called a Hash value or Hash using various methods like Truncate, Division Reminder, Mid Square Method and Folding Method etc. Using these functions we calculate a Hash value which represents the original value, but it is normally smaller than the original.

In some cases, there are chances of same hash values is calculated for same memory location. It means collisions occur when the hash function maps two different keys to the same location. Obviously, two records

cannot be stored in the same location. Therefore, a method used to solve the problem of collision, also called collision resolution technique, is applied. The two most popular methods of resolving collisions are Collision Resolution by Separate Chaining (Open Hashing) and Collision Resolution by Open Addressing (Closed Hashing)

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Self - Evaluation

1. What do you mean by hash table?
2. What is hash function? What are the qualities of a good hash function?
3. Write a short note on the different hash functions. Give suitable examples to justify your answers.
4. Calculate hash values of keys: 8922, 9241, 3807 , 5643 and 4522 using different methods of hashing.
5. What is collision? How to resolve a collision. Which technique do you think is better and why?
6. Consider a hash table with size = 10. Using linear probing, insert the keys 72, 27, 33, 54, 26, 68, 59,and 101 into the table.
7. What is hashing? Give its applications. Also, discuss the pros and cons of hashing.
8. Explain chaining with examples.
9. Write short notes on:
    - Mid square Method

- Folding Method
- Modular Method
- Linear probing
- Quadratic probing
- Double hashing

10. What are applications of hashing?

MCQ :

1. What is a hash table?

   a) A structure that maps values to keys

   b) A structure that maps keys to values

   c) A structure used for storage

   d) A structure used to implement stack and queue

2. If several elements are competing for the same memory location in the hash table, what is it called?

   a) Diffusion

   b) Replication

   c) Collision

   d) None of the mentioned

3. What is direct addressing?

   a) **Distinct array position for every possible key**

   b) Fewer array positions than keys

   c) Fewer keys than array positions

   d) None of the mentioned

4. What can be the techniques to avoid collision?

   a) Make the hash function appear random

   b) Use the chaining method

   c) Use uniform hashing

   d) All of the mentioned

5. What is a hash function?

   a) A function has allocated memory to keys

   b) A function that computes the location of the key in the array

c) A function that creates an array

d) None of the mentioned

UNIT-12 FILE STRUCTURE

Structure

12.0 INTRODUCTION

File organization refers to the way data is stored in a file. File organization is very important because it determines the methods of access, efficiency, flexibility and storage devices to use in efficient way.

Nowadays, almost all organizations use data collection software to collect large amounts of data. For example, when we approach any college for admission, our all data like name, address, phone number, the requested course, aggregate of marks obtained in the last examination etc. is collected.

A university might like to store data related to all students—the courses they sign up for etc., all this implies the following:

- Data will be stored on external storage devices like magnetic tapes, disk, floppy etc.
- Data will be accessed by many people and software programs
- Users of the data will expect that
    - It is always reliably available for processing
    - It is secure
    - It is stored in a manner flexible enough to allow the users to add new data as per changing needs

In common terminology, a file is a block of important data which is available to any computer software and is usually stored on any storage device. Storing a file on any storage medium like pen drive, hard disk or floppy disk ensures the availability of the file in future.Now a days all file

are stored in computers to reduce paper work and easy availability in any office, bank or library.

12.1 OBJECTIVES

After reading this unit the learner is able to do the following task.

- Understand the concept of File and its terminology.
- File Organization and its uses
- Operations that could be performed of file
- Sequential File organization and its features
- Direct File organization and its features
- Index Sequential File organization and its features

12.2 FILE TERMINOLOGY

Every file contains records which could be of a customer (in Business), students (in university or college), patient (in hospital), passenger (in train/flight reservation) which can be organized in a hierarchy to present a systematic way so that it could be utilized in future as requested.

Now we will define the terms of the hierarchical structure of data which we are storing in computer in the form of file.

Field : It is an elementary data item that stores a single fact and characterized by its length and types.

For example :

Name :	Size= 25	Type= Character
Age :	Size=2	Type= Integer/Numeric
Address:	Size=100	Type=Character

Record : It is a collection of related fields that can be treated as a unit from an applications point of view.

For example : The student's record may contain data fields such as name, address, phone number, roll number, marks obtained, and so on.

File : Data is organized for storage in files. A file is a collection of similar, related records. It has an identifying name.

For example : There are 100 students in a class, then there are 100 records, one for each student. All these related records are stored in a file with the respective class name.

Directory : A directory stores information of files belongs to one group or related in any manner. A directory is used to store files so that users can access all files easily.

For example :

Teachers : This directory will stores all files of teachers.

BSc_Part1 : This directory will stores all first year students.

Courses : This directory will stores all information of available courses.

12.3 FILE ORGANIZATION

File organization tell that what is the way to store data and how to retrieve that data. Basically File Organization is a way of arranging the records in a file when the file is stored on the disk. The factors involved in selecting a particular file organization for uses are:

- Easily storing and retrieval of information
- Easily modify or updates
- Less storage space
- Reliability of data for future
- Security
- Integrity

Different file organizations assures the above factors with different weightages. The choice of particular file organization depends on the type of application and need of the user.

Now we will discuss some of the file organization in brief.

Sequential Files : Data records are stored in some specific sequence e.g., order of arrival, value of key field etc. Records of a sequential file cannot be accessed at random i.e., to access the nth record, one must traverse the preceding (n - 1) records. Sequential files will be dealt with at length in the next section.

Relative Files : Each data record has a fixed place in a relative file. Each record must have associated with it an integer key value that will help identify this slot. This key, therefore, will be used for insertion and retrieval of the records. Random as well as sequential access is possible. Relative files can exist only on random access devices like disks.

Direct Files : These are similar to relative files, except that the key value need not be an integer. The user can specify keys which make sense to his application.

Indexed Sequential Files : An index is added to the sequence file to provide random access. An overflow area needs to be maintained to permit insertion in sequence.

12.4 BASIC FILE OPERATIONS

Before moving to a particular organization, let usunderstand basic file operations that can be performed on any kind of file.

A. Creation of file

B. Updation of file

   I. Insert a new record

   II. Modify an existing record

   III. Delete a particular record

C. Retrieval of Information

   I. Inquiry of record

   II. Generate a report

D. Maintenance

   I. Restructuring the file records

   II. Reorganizing the records

A. Creation of File:

Before creating a file we have to collect the data, validate the data and process the data to give it a record like structure. Then we choose the name for the file and open the file on secondary storage device and store the collected data on it.

B. Updating File:

It means to change/delete/modify the contents of the file. A file can be updated in the following ways:

- **Inserting a new record**: For example, if a new student comes later and joins the course, we have to add his record in the STUDENT file.
- **Modifyingan existing record**: For example, if the name of a student was spelt incorrectly, then correcting the name will be a modification of the existing record.
- **Deletingan existing record**. For example, if a student switch or quits a particular course in the initial/middle of the session, his/her record has to be deleted from the STUDENT file.

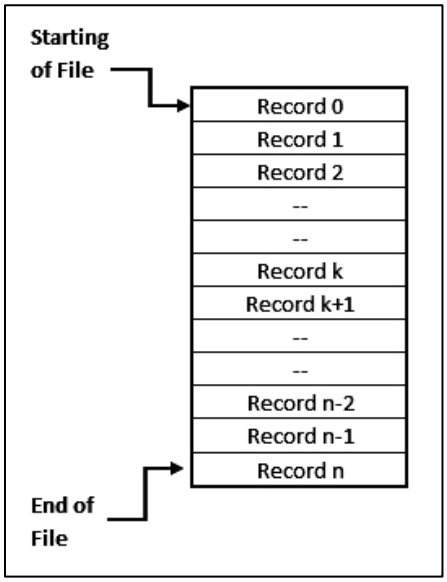
C. Retrieving from a File :

It means, we are accessing only the useful data from a given file as per the requirement. Information can be retrieved for an inquiry or for report generation. For any inquiry less data is retrieved while to generate a report needs large amount of data from the file.

D. Maintenance of File:

It involves restructuring or re-organizing the file to improve the performance of the programs that access this file. Restructuring a file keeps the file organization unchanged and changes only the structural aspects of the file (for example, changing the field width or adding/deleting fields). On the other hand, file reorganization may involve changing the entire organization of the file. We will discuss file organization in detail in the next section.

12.5 SEQUENTIAL FILE



Sequential files have data records stored in a specific sequence. A sequential organized file may be stored on either a serial access or a direct access storage medium.

A sequentially organized file stores the records in the order in which they were entered. That is, the first record that was entered is written as the first record in the file, the second record entered is written as the second record in the file, and so on. As a result, new records are added only at the end of the file. Sequential files can be read only sequentially, starting with the very first record in the file to the last record of the file. It is the most basic and simple file organization to organize a large collection of records in a file.

Once we store the records in a file, we cannot modify the records means deletion or Updation in sequential file is not allowed. You have to create a new file with existing file to do any such operation like delete or modify.

All records have the same size and the same field format. The records are sorted based on the value of one field or a combination of two or more fields. Records can be sorted in either ascending or descending order.

These files are used to generate reports or to perform sequential reading of large amount of data which some programs need to do such as payroll

processing, or billing of all customers. Sequential files can be easily stored on both disks and tapes.

The features, advantages, and disadvantages of sequential file organization are as under.

Features

- Records are written in the order in which they are entered
- Records are read and written sequentially
- Deletion or Updation of one or more records calls for replacing the original file with a new file that contains the desired changes
- Records have the same size and the same field format
- Records are sorted on a key value
- Generally used for report generation or sequential reading

Advantages :

- Simple and easy to handle
- No extra overheads involved
- Sequential files can be stored on magnetic disks as well as magnetic tapes
- Well suited for batch oriented applications

Disadvantages :

- All records must be structurally identical. If a new field has to be added, then every record must be rewritten to provide space for the new field.
- Records can be read only sequentially. If i^{th} record has to be read, then all the i–1 records must be read
- Updates are not easily accommodated. Does not support update operation? A new file has to be created and the original file has to be replaced with the new file that contains the desired changes
- By definition, random access is not possible
- Continuous areas may not be possible because both the primary data file and the transaction file must be looked during merging.

Areas of Use :

Sequential files are most frequently used in commercial batch oriented data processing where there is the concept of a master file to which details are added periodically. For example, Payroll applications.

Example: Write a program to implement sequential file operation (Write, Read) using C.

```
#include <stdio.h>
#include <conio.h>

typedefstruct {
        intusn;
        char name[25];
        int m1,m2,m3;
   }Student;
Student s;
void display(FILE *);
int search(FILE *,int);
void main()
 {      inti,n,usn_key,opn;
        FILE *fp;
        printf(" How many Records ? ");
        scanf("%d",&n);
        fp=fopen("stud.dat","w");
        for (i=0;i<n;i++) {
                printf("Read the Info for Student: %d
(usn,name,m1,m2,m3) \n",i+1);

        scanf("%d%s%d%d%d",&s.usn,s.name,&s.m1,&s.m2,&s.m3);
                fwrite(&s,sizeof(s),1,fp);
        }
        fclose(fp);
        fp=fopen("stud.dat","r");
do {
        printf("Press 1- Display\t 2- Search\t 3- Exit\t Your Option?");
        scanf("%d",&opn);
        switch(opn)
        {
        case 1: printf("\n Student Records in the File \n");
                display(fp);
                break;
        case 2: printf(" Read the USN of the student to be searched ?");
                scanf("%d",&usn_key);
                if(search(fp,usn_key))
                {printf("Success ! Record found in the file\n");

        printf("%d\t%s\t%d\t%d\t%d\n",s.usn,s.name,s.m1,s.m2,s.m3);
        }
 else
                printf(" Failure!! Record with USN %d not
found\n",usn_key);
                break;
        case 3: printf(" Exit!! Press a key . . .");
```

```
                getch();
                break;
        default:  printf(" Invalid Option!!! Try again !!!\n");
                break;
        }
        }while(opn != 3);
        fclose(fp);
   }
   /* End of main() */
   void display(FILE *fp) {
        rewind(fp);
        while(fread(&s,sizeof(s),1,fp))
        printf("%d\t%s\t%d\t%d\t%d\n",s.usn,s.name,s.m1,s.m2,s.m3);
   }
int search(FILE *fp, intusn_key)
{
        rewind(fp);
        while(fread(&s,sizeof(s),1,fp))
          if( s.usn == usn_key) return 1;
        return 0;
}
```

3.6 DIRECT FILE

In direct file organization the key value is mapped directly or indirectly to a storage location, avoiding the use of indices. The usual method of direct mapping is by some arithmetical manipulation of the key value, also known as hashing. It offers an effective way to organize data when there is a need to access individual records directly. A calculation is performed on the key value to get an address. This address calculation technique is often termed as hashing. The calculation applied is called a hash function.

Record No	Record in Memory
0	Record 0
1	Record 1
2	Record 2
3	Record 3
4	Record 4
5	Record 5
6	Record 6
--	--
--	--
999	Record 999
1000	Record 1000

Direct file organization provides random access by directly jumping to the record which has to be accessed. If the records are of fixed length and we know the base address of the file and the length of the record, then any record i can be accessed using the following formula:

*Address of n^{th} Record = Starting_address_of_file + (n–1) * Record_Size*

Therefore, in direct files, records are organized in ascending relative record number. A direct file can be thought of as a single dimension table stored on a disk, in which the relative record number is the index into the table. Direct files can be used for both random as well as sequential access. For sequential access, records are simply read one after another.

Features

- Provides an effective way to access individual records
- The record number represents the location of the record relative to the beginning of the file
- Records in a relative file are of fixed length
- Relative files can be used for both random as well as sequential access
- Every location in the table either stores a record or is marked as FREE

Advantages of Direct File Organization

1. Records can be immediately accessed for Updation.
2. Several files can be simultaneously updated during transaction processing.
3. Transaction need not be sorted.
4. Existing records can be amended or modified.
5. Most suitable for interactive online applications.
6. Very easy to handle random enquiries.

Disadvantages of Direct File Organization

1. Data may be accidentally erased or overwritten unless special precautions are taken.
2. Risk of loss of accuracy and breach of Security.
3. Special backup and reconstruction procedures must be established.
4. Expensive hardware and Software are required.
5. High complexity in programming.
6. Use of relative files is restricted to disk devices
7. Records can be of fixed length only
8. For random access of records, the relative record number must be known in advance

Example : Write a program to implement Directfile operation (Write, Reading record randomly) using C.

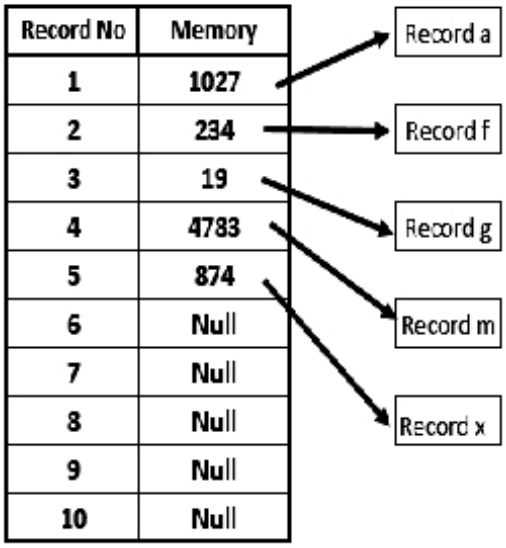
```
/* Program to save alphabet A to Z in file and then print the letters
sequentially or randomly*/

#include <stdio.h>
int main(void)
{
inti;
      char ch;
      FILE *fptr;
clrscr();
fptr = fopen("char", "w");

  if (fptr != NULL)
          printf("File created successfully!\n");
  else
      {
printf("Failed to create the file.\n");
       return -1;
      }
  // writing all characters in data file
  for (ch = 'A'; ch<= 'Z'; ch++)
putc(ch, fptr);
fclose(fptr);

printf("\nCounting of Characters :\n");
  for (i = 0; i< 26; i++)
printf(" %2d", (i+1));

printf("\n");

  for (i = 65; i<= 90; i++)
          printf("%3c", i);

printf("\n\n");

  // Again open file for reading
fptr = fopen("char", "r");
printf("Currpos: %ld\n", ftell(fptr));

  // read 1st char in the file
fseek(fptr, 0, 0);
ch = getc(fptr);
printf("1st char: %c\n", ch);
printf("Currpos: %ld\n", ftell(fptr));

  // read 5th char in the file
```

```
fseek(fptr, 4, 0);
ch = getc(fptr);
printf("5th char: %c\n", ch);
printf("Currpos: %ld\n", ftell(fptr));

fseek(fptr, 25, 0);        // read 26th char in the file
ch = getc(fptr);
printf("26th char: %c\n", ch);
printf("Currpos: %ld\n", ftell(fptr));
printf("Rewind : Moving to first location in the file\n");
  rewind(fptr);
printf("Currpos: %ld\n", ftell(fptr));
fseek(fptr, 9, 0);           // read 10th char in the file
ch = getc(fptr);
printf("10th char: %c\n", ch);
printf("Currpos: %ld\n", ftell(fptr));

  // read 15th char in the file
fseek(fptr, 4, 1);    // move 4 bytes forward from current position
ch = getc(fptr);
printf("15th char: %c\n", ch);
printf("Currpos: %ld\n", ftell(fptr));

  // read 20th char in the file
fseek(fptr, 4, 1);    // move 4 bytes forward from current position
ch = getc(fptr);
printf("20th char: %c\n", ch);

printf("Currpos: %ld\n", ftell(fptr));

fclose(fptr);
  return 0;
}
```

3.7 Indexed Sequential Files

On an average, the retrieval of a record from a sequential file, requires access to half the records in the file. It makes it not only inefficient but very time consuming process for the large file. So to improve the query response time from a sequential file, a type of indexing technique can be added.When there is need to access records sequentially by some key value and also to access records directly by the same key value, the collection of records may be organizedin an effective manner called Indexes Sequential Organization.

Indexed sequential file organization stores data for fast retrieval. The records in an indexed sequential file are of fixed length and every record is uniquely identified by a key field. We maintain a table known as the index table which stores the record number and the address of all the records. That is for every file, we have an index table. This type of file organization is called as indexed sequential file organization because physically the records may be stored anywhere, but the index table stores the address of those records.



A sequential (or sorted on primary keys) file that is indexed is called an Index Sequential File. The index provides for random access to records, while the sequential nature of the file provides easy access to the subsequent records as well as sequential processing. An additional feature of this file system is the overflow area. This feature provides additional space for record addition without necessitating the creation of a new file.

For example, Let us take an example of a school where the details of students are stored in an indexed sequential file. Now we can accessed the records from this file in two different ways:

- Sequentially : To print the report card of each student in an exam
- Randomly : To modify the marks of a particular student in any subject that are wrongly typed by typist.

Advantages of Indexed sequential access file organization

- In indexed sequential access file, sequential file and random file access is possible.
- It accesses the records very fast if the index table is properly organized.
- The records can be inserted in the middle of the file.
- It provides quick access for sequential and direct processing.
- It reduces the degree of the sequential search.

Disadvantages of Indexed sequential access file organization

- Indexed sequential access file requires unique keys and periodic reorganization.
- Indexed sequential access file takes longer time to search the index for the data access or retrieval.
- It requires more storage space.
- It is expensive because it requires special software.
- It is less efficient in the use of storage space as compared to other file organizations.

3.8 SUMMARY

A file is a collection or bag of records. Having stored the records in a file, it is necessary to access these records using either a primary or secondary key. The type and frequency of access required determines the type of file organization to be used for a given set of records. In this chapter we looked at some common file organizations: Sequential, Index sequential, direct etc.

In a sequential file, records are maintained in the logical sequence of their primary key value. The search for a given record requires, on average, access to half the records in the file. Update operations, including the appending of a new record, require creation of a new file. Updates could be batched and a transaction file of updates used to create a new master file from the existing one. This scheme automatically creates a backup copy of the file.

Access to a sequential file can be enhance by creating an index. The index provides random access to records and the sequential nature of the file provides easy access to the next record. To avoid frequent reorganization, an index sequential file uses overflow areas. This scheme provides space for the addition of records without the need for the creation of a new file. In index sequential organization, it is the usual practice to have a hierarchy of indexes with the lowest level index pointing to the records while the higher level ones point to the index below them.

In direct file organization the key value is mapped directly or indirectly to a storage location, avoiding the use of indices. The usual method of direct mapping is by some arithmetical manipulation of the key value, the process is called hashing.

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SELF EVALUATION

Short Answer Questions

1. Why do you mean by file concept?
2. What is the need of file in computer system?
3. Explain the terms field, record, directory and index.
4. What are various operations that you can perform on file?
5. What do you understand by the File organization? Discuss in brief with suitable example.
6. What is Sequential file organization and its advantage?
7. Differentiate between Sequential file organization and Direct file organization.
8. What is Index Sequential file organization and its advantages?

Multiple Choice Questions

1. A …….. is the basic element of data where individual field contains a single value, such as an employee's last name or DOB or Mob No.

   A) Field B) record

   C) file D) database

2. A ……is collection of related fields that can be treated as a unit by some application program.

   A) Field **B) record**

   C) file D) database

3. In ……file organization, a fixed format is used for records where all records are of the same length, consisting of the same number of fixed length fields in a particular order.

   A) Flat File **B) sequential**

   C) indexed sequential D) indexed

4. The …… maintains the key characteristic of the sequential file: Records are organized in sequence based on a key field.

   A) Flat File B) sequential file

   C) indexed sequential file D) indexed file

5. The ……. greatly reduced the time required to access a single record, without sacrificing the sequential nature of the file.

A) Flat File B) sequential file

C) indexed sequential file D) indexed file

6. An alternative is to organize the sequential file physically is a ……..

A) List **B) Linked List**

C) Queue D) Stack

7. ……. are typically used in batch applications and are generally optimum for such applications if they involve the processing of all the records.

A) Indexed files B) Direct files

C) Sequential files D) Indexed Sequential files

8. Which is the simplest file structure.

A) Indexed files B) Direct files

C) Sequential files D) Flat File